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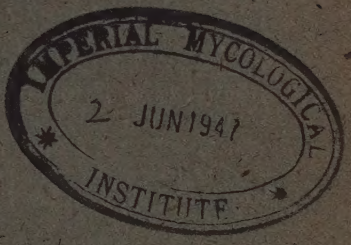

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MATHEMATICS

ON THE CONVERGENCE OF THE METHOD OF REDUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

By L. KANTOROVITCH

(Communicated by S. N. Bernstein, Member of the Academy, 25. XI. 1940)

In a previous paper of mine (1) were given sufficient conditions for the convergence of a minimal sequence to the solution of a partial differential equation. It was shown that it is important for the convergence of a minimal sequence that the expression $\varepsilon_n = I(u_n) - I(u) = I_1(u_n - u)$ diminishes sufficiently rapidly. In this paper we shall show for the cases of the method of reduction to ordinary differential equations [proposed in 1932 by the author of the present paper (2)], under what conditions can we be sure that ε_n diminishes with sufficient rapidity.

For simplicity's sake we shall consider only the domains of simplest shape.

Lemma 1. Let D be a domain bounded by the lines: $x=0$; $x=l$; $y=g(x)$; $y=h(x)$, where the functions $g(x)$ and $h(x)$ are defined in the segment $0 \leqslant x \leqslant 1$ and possess in this segment bounded derivatives of the first order; let, besides, in the same segment $h(x) > g(x)$. Let further $u(x, y)$ be a function defined in D, vanishing on the boundary of D and such that the functions $\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial^2 u}{\partial x \partial y}$ exist and are integrable with their squares in D.

Then functions $\bar{u}_n(x, y)$ and $\tilde{u}_n(x, y)$ can be constructed $(n = 1, 2, \ldots)$ such that they have the form

$$\left.\begin{aligned} \bar{u}_n(x, y) &= \sum_{k=1}^{n} f_k(x) \sin \frac{k\pi (y - g(x))}{h(x) - g(x)} \\ \tilde{u}_n(x, y) &= \sum_{k=1}^{n} f_k(x)\, y^{k-1} (y - g(x))\,(y - h(x)) \end{aligned}\right\} \qquad (1)$$

$f_k(0) = f_k(l) = 0$, and that the quantities $\bar{\varepsilon}_n = I(\bar{u}_n) - I(u)$ and $\tilde{\varepsilon}_n = I(\tilde{u}_n) - I(u)$ have the order $0\left(\frac{1}{n^2}\right)$.

Proof. We shall begin with $\bar{u}_n(x, y)$. We set:

$$v(x, z) = u\left(x, g + \frac{z}{\pi}(h - g)\right);\ 0 \leqslant z \leqslant \pi;\ 0 \leqslant x \leqslant 1 \qquad (2)$$

$$\left.\begin{aligned} \sigma(x, z) &= \sum_{k=1}^{n} a_k \sin kz; \quad a_k = \frac{2}{\pi} \int_0^{\pi} v(x, z) \sin kz\, dz, \\ \rho(x, z) &= v(x, z) - \sigma(x, z). \end{aligned}\right\} \qquad (3)$$

Then integrating by parts and taking into consideration that $v=0$ for $z=0$ and $z=\pi$, we find:

$$\left.\begin{aligned}\frac{\partial \rho}{\partial z}=\frac{\partial v}{\partial z}-\sum_{k=1}^{n} ka_k\cos kz=\frac{\partial v}{\partial z}-\sum_{k=1}^{n}\left(\frac{2k}{\pi}\int_0^{\pi} v(x,z)\sin kz dz\right)\cos kz=\\ =\frac{\partial v}{\partial z}-\sum_{k=1}^{n}\left(\frac{2}{\pi}\int_0^{\pi}\frac{\partial v}{\partial z}\cos kz dz\right)\cos kz.\end{aligned}\right\} \tag{4}$$

Hence

$$\left.\begin{aligned}\frac{2}{\pi}\int_0^{\pi}\left(\frac{\partial \rho}{\partial z}\right)^2 dz=\sum_{k=n+1}^{\infty}\left(\frac{2}{\pi}\int_0^{\pi}\frac{\partial v}{\partial z}\cos kz dz\right)^2=\\ =\sum_{k=n+1}^{\infty}\left(\frac{2}{k\pi}\int_0^{\pi}\frac{\partial^2 v}{\partial z^2}\sin kz dz\right)^2\leqslant\frac{1}{(n+1)^2}\frac{2}{\pi}\int_0^{\pi}\left(\frac{\partial^2 v}{\partial z^2}\right)^2 dz\\ \int_0^{l} dx\int_0^{\pi}\left(\frac{\partial \rho}{\partial z}\right)^2 dz\leqslant\frac{1}{(n+1)^2}\int_0^{l} dx\int_0^{\pi}\left(\frac{\partial^2 v}{\partial z^2}\right)^2 dz.\end{aligned}\right\} \tag{5}$$

It remains to prove that the integral in the right-hand member of (5) is finite; we shall prove this a little later.

Similarly to (5) we find

$$\int_0^{l} dx\int_0^{\pi}\left(\frac{\partial \rho}{\partial x}\right)^2 dz\leqslant\frac{1}{(n+1)^2}\int_0^{l} dx\int_0^{\pi}\left(\frac{\partial^2 v}{\partial x\partial z}\right)^2 dz \tag{6}$$

$$\int_0^{l} dx\int_0^{\pi}\rho^2 dz\leqslant\frac{1}{(n+1)^4}\int_0^{l} dx\int_0^{\pi}\left(\frac{\partial^2 v}{\partial z^2}\right)^2 dz; \tag{7}$$

combining (5), (6) and (7) we obtain

$$\int_0^{l}\int_0^{\pi}\left[\left(\frac{\partial \rho}{\partial x}\right)^2+\left(\frac{\partial \rho}{\partial z}\right)^2+\rho^2\right]dxdz<\frac{C_1}{(n+1)^2}. \tag{8}$$

We shall introduce now the variable y defining it as follows: $z=\frac{\pi(y-g)}{h-g}$, and set

$$S(x,y)=\sigma\left(x,\frac{\pi(y-g)}{h-g}\right)=\sum_{k=1}^{n} a_k\sin\frac{k\pi(y-g)}{h-g} \tag{9}$$

$$R(x,y)=u(x,y)-S(x,y)=\rho\left(x,\frac{\pi(y-g)}{h-g}\right). \tag{10}$$

If we now write down the expressions of derivatives of u and R, then, according to suppositions that the integrals of $\left(\frac{\partial^2 u}{\partial x\partial y}\right)^2$ and $\left(\frac{\partial^2 u}{\partial y^2}\right)^2$ are finite and $h'(x)$ and $g'(x)$ are bounded, we can see that the integrals on the right-hand sides of (5), (6), (7) are finite, which completes the proof of the inequality (8). But now substituting in (8) new variables we get the inequality

$$\iint_D\left[\left(\frac{\partial R}{\partial x}\right)^2+\left(\frac{\partial R}{\partial y}\right)^2+R^2\right]dxdy<\frac{C_2}{(n+1)^2}. \tag{11}$$

Finally, taking into consideration that the functions a, b and c are bounded, we get

$$I_1(R) = \iint_D \left[a\left(\frac{\partial R}{\partial x}\right)^2 + b\left(\frac{\partial R}{\partial y}\right)^2 + cR^2 \right] dxdy < \frac{C_3}{(n+1)^2}. \qquad (12)$$

This last inequality, together with (9) and (10), shows that we can take $\bar{u}_n(x, y) = S(x, y)$, and then we shall have $\bar{\varepsilon}_n < C_3 : (n+1)^2$, i. e. $\bar{\varepsilon}_n = 0(n^{-2})$. Besides it follows immediately from (3) that $a_k(0) = a_k(1) = 0$ and therefore $f_k(0) = f_k(1) = 0$.

We shall pass now to the functions $\tilde{u}_n(x, y)$. Let us set $\cos\zeta = 4\left(\frac{z}{\pi}\right)^3 - 6\left(\frac{z}{\pi}\right)^2 + 1$ for $0 \leqslant z \leqslant \pi$. Then for $0 \leqslant \zeta \leqslant \pi$ we have defined a function $z = \varkappa(\zeta)$, having the derivatives of all orders and such that $\varkappa'(\zeta) \neq 0$ for $0 \leqslant \zeta \leqslant \pi$. Let further

$$w(x, \zeta) = \frac{\sin\zeta}{z(\pi - z)} v(x, z); \quad z = \varkappa(\zeta); \quad 0 \leqslant \zeta \leqslant \pi. \qquad (13)$$

Here the expression $\frac{\sin\zeta}{z(\pi - z)}$ possesses a derivative for $0 \leqslant \zeta \leqslant \pi$. Let us set finally

$$\tau(x, \zeta) = \sum_{k=1}^{n} b_k \sin k\zeta; \quad b_k = \frac{2}{\pi}\int_0^{\pi} w(x, \zeta) \sin k\zeta d\zeta. \qquad (14)$$

Then, in a similar way as we have proved (8), we can show that

$$\int_0^l dx \int_0^{\pi} \left[\left(\frac{\partial w}{\partial x} - \frac{\partial \tau}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial \zeta} - \frac{\partial \tau}{\partial \zeta}\right)^2 + (w - \tau)^2 \right] d\zeta < \frac{C_4}{(n+1)^2}. \qquad (15)$$

We have further

$$\tau(x, \zeta) = \sin\zeta\, P_{n-1}(\cos\zeta)$$

where P_{n-1} is a polynomial of $(n-1)$-th degree. Substituting for $\cos\zeta$ its expression in z, we obtain

$$w(x, \zeta) = \frac{\sin\zeta}{z(\pi - z)} T(x, z), \text{ where } T(x, z) = z(\pi - z)\, P_{n-1}(\cos\zeta) =$$
$$= z(\pi - z)\, P_{3n-3}(z). \qquad (16)$$

If we now substitute in (15) for w its expression in v (13) and for τ its expression in T (16), we obtain

$$\int_0^l dx \int_0^{\pi} \left[\left(\frac{\partial v}{\partial x} - \frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} - \frac{\partial T}{\partial z}\right)^2 + (v - T)^2 \right] dz < \frac{C_5}{(n+1)^2}. \qquad (17)$$

Returning to our old variable y, we can take for u_n the following expressions

$$\tilde{u}_{3n-2}(x, y) = \tilde{u}_{3n-1}(x, y) = \tilde{u}_{3n}(x, y) = T\left(x, \pi\frac{y - g(x)}{h(x) - g(x)}\right). \qquad (18)$$

Now it can be easily seen (16) that the functions u_n are of the required form (1) and also that

$$\tilde{\varepsilon}_n = I_1(u - \tilde{u}_n) = \iint_D \left[a\left(\frac{\partial u}{\partial x} - \frac{\partial \tilde{u}_n}{\partial x}\right)^2 + \right.$$
$$\left. + b\left(\frac{\partial u}{\partial y} - \frac{\partial \tilde{u}_n}{\partial y}\right)^2 + c(\tilde{u}_n - u)^2 \right] dxdy < \frac{C_6}{n^2} \qquad (19)$$

which proves our lemma. [In the proof of (19) we use the inequality (17) and the proof itself is exactly similar to that of (12)].

Remark 1. The conditions of the lemma can be a little weakened. Viz., we require that Liouville's derivatives $\frac{\partial^{1+p}u}{\partial x \partial y^p}$ and $\frac{\partial^{1+p}u}{\partial y^{1+p}}$ (for $0 < p < 1$) be integrable together with their squares; then we can prove that $\varepsilon_n = \tilde{\varepsilon}_n = 0(n^{-2p})$. The proof will then remain the same, we must only use a theorem of Hardy and Littlewood ([3]) which states that if a function $f(x)$ has a derivative of p-th order integrable in $[0, \pi]$ together with its square, then

$$\int_0^{\pi} (f - S_n)^2 \, dx = 0\left(\frac{1}{n^{2p}}\right)$$

if S_n is the n-th Fourier sum for $f(x)$.

From this lemma and Theorem 2 of the paper ([1]) referred to above follows immediately

Theorem 1. *If D is a region of the form specified in Lemma 1 and $u(x, y)$ is a function furnishing the solution of a boundary problem for the equation of elliptic type, more precisely: if u satisfies the conditions of Theorem 1 of the paper ([1]) and if for u the conditions of Lemma 1 are fulfilled, then the sequence of approximations found by the method of reduction to ordinary equations converges uniformly towards a solution, provided that we seek those approximations in one of the forms (1) with indefinite functions $f_k(x)$. Besides, the order of the error of n-th approximation is* $0\left(\frac{\sqrt{\lg n}}{n}\right)$.

Remark 2. Note that on the assumption of the integrability with squares of the derivatives of u of $(s+1)$-th order, we can find a more precise estimation for the error, viz.

$$|u_n - u| = 0\left(\frac{\sqrt{\lg n}}{n^s}\right). \qquad (20)$$

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MATHEMATICS

GENERALIZED DUALITY THEOREM FOR COMMUTATIVE GROUPS WITH AN INVARIANT MEASURE

By D. RAIKOV

(Communicated by A. N. Kolmogoroff, Member of the Academy, 27. XII. 1940)

Let us suppose that G is a commutative topological group satisfying the second axiom of countability*; (B) is the least Borel's field containing all open sets from G; and on (B) there is defined a totally additive non-negative measure $\mu(E)$ $(E \in (B))$, having the following properties:

1. *Non-triviality:* there exists a neighbourhood of zero with a finite positive measure.

2. *Invariance:* $\mu(E+g)=\mu(E)$ for any $E \in (B)$ and every $g \in G$.

It may be shown then, that μ has the following properties:

3. *Regularity:* $\mu(E)=\inf \mu(U)$, where U are open sets which cover E.

4. *Continuity:* if $\mu(E)<\infty$, then for any $\varepsilon>0$ there exists such a neighbourhood U_ε of zero that $\mu(E)-\mu(E \cap E+g)<\varepsilon$ for every $g \in U_\varepsilon$.

5. *Inverse invariance:* $\mu(-E)=\mu(E)$.

The last property may be obtained simultaneously with the *uniqueness* of measure in a very simple manner (which cannot be used, however, in the case of non-commutative groups).

By $L_p(G)$ we shall denote the space of measurable functions $x(g)$, for which $\|x\|^p=\int |x(g)|^p dg<\infty$. The functions of the space $L_1(G)$ form a ring with the usual addition and with «convolution» as multiplication. In [1] there was considered a normed ring R which coincided with $L_1(G)$, when G was discrete [so that $L_1(G)$ contained a unit with respect to convolution], and which was obtained by the formal adjoining of the unit to $L_1(G)$, when G was not discrete [so that in the last case $L_1(G)$ formed in R a maximal ideal]. It was shown that the set X of all continuous characters of G might be considered as the set of all maximal ideals of the ring R different from $L_1(G)$ (hence, when G was discrete, as a set of all maximal ideals without exception). Transferring into X the topology from the space of maximal ideals, we obtain a locally bicompact space (which is bicompact when G is discrete). Hereby the neighbourhoods of continuous character χ_0 are defined by arbitrary finite sets of functions $x_k(g) \in L_1(G)$ $(k=1, \ldots, n)$ and arbitrary numbers $\varepsilon>0$, as the totalities of all continuous characters χ satisfying the inequalities

$$\left|\int x_k(g)\chi(g)\,dg-\int x_k(g)\chi_0(g)\,dg\right|<\varepsilon \quad (k=1, \ldots, n). \tag{1}$$

* All results hold true, when instead of the second axiom of countability, the local bicompactness of G is supposed (the invariant measure exists, as is known, in all locally bicompact groups, independently of whether they satisfy the second axiom of countability, or not).

X naturally forms a group where the product of characters χ_1 and χ_2 is defined as a character

$$\chi_1\chi_2(g) = \chi_1(g)\,\chi_2(g). \tag{2}$$

Theorem 1. *X is a locally bicompact topological group. If on G the second axiom of countability holds true, it does so on X as well. If G is locally bicompact, the topology in X defined by neighbourhoods* (1) *coincides with the topology introduced by L. Pontrjagin* ([2], Definition 34).

In virtue of the well-known result by Haar ([3]) there exists on X an invariant measure. By $L_p(X)$ we shall denote the space of functions $a(\chi)$ measurable with respect to this measure, for which

$$\|a\|^p = \int |a(\chi)|^p\, d\chi < \infty.$$

M. Krein ([4]) has shown that between the spaces $L_2(G)$ and $L_2(X)$ there exists a connexion given by the «generalized Plancherel's theorem», according to which *the operator* $Px = \int x(g)\,\chi(g)\,dg$ $(x(g) \in L_2(G))$ *maps* $L_2(G)$ *on* $L_2(X)$ *isometrically, and the operator* $Qa = \int a(\chi)\,\overline{g}(\chi)\,d\chi (a(\chi) \in L_2(X))$ *is the inverse to it**.

In the present note this theorem will serve to establish the generalized duality theorem for commutative groups having an invariant measure (which includes Pontrjagin's duality theorem for locally bicompact groups).

In virtue of (2) every element of G is, in its turn, a (continuous) character of X. Since G possess a sufficient set of continuous characters ([5], [2], [1]) **, the different elements of G represent the different characters of X. Thus, G is contained in the group G^* of all continuous characters of X. Let us topologize G^* as a set of maximal ideals of the ring R' formed from $L_1(X)$ in the same way as R is formed from $L_1(G)$ (or, which amounts to the same according to theorem 1, introduce on G^* Pontrjagin's topology).

Theorem 2 (the generalized duality theorem). *G is the everywhere dense homeomorph part of G^*.*

Proof. We shall use the following notation: by (x, χ) $(x(g) \in L_1(G), \chi \in X)$ we shall denote the integral $\int x(g)\,\chi(g)\,dg$; by (a, g^*) $(a(\chi) \in L_1(X), g^* \in G^*)$ the integral $\int a(\chi)\,g^*(\chi)\,d\chi$; by f_E the characteristic function of the set E. The Latin capitals with asterisk will denote the sets from G^*, those without asterisk the projections of these sets in G. The group operation in G^* will be written additively.

1°. *G is a homeomorph part of G^*.* Indeed, the sets U^* of the form

$$U^* = \{|(a_k, g^*) - (a_k, g_0^*)| < \varepsilon\} \quad (k = 1, \ldots, n; \quad a_1, \ldots, a_n \in L_1(X))$$

represent in G^* the neighbourhoods of the element g_0^*. But the functions (a, g), where $a \in L_1(X)$, $g \in G$, are continuous in G, as follows from the Lemma referred to in ([6]) and from the properties of Haar's measure on the locally bicompact groups. Therefore the projection U of any neighbourhood $U^* \subset G^*$ is an open set in G. We have to show yet, that for each neighbourhood V of the zero of G such a neighbourhood U^* exists, that $U \subset V$. Let W be a neighbourhood of the zero of G, having a finite measure

* An other proof of this theorem of M. Krein, based on the same conception, but in which the instrument of positive definite functions was used in a more consecutive way, had already been found by me in course of my attempts to prove the generalized duality theorem, after I had been informed by M. Krein about his result. The method employed by M. Krein was unknown to me at that time.

** See also ([6]), where one more proof of this theorem is implicitely contained.

and such that $W+(-W)\subset V$. Put $\varphi(g)=\int f_W(g+h)\,f_W(h)\,dh$. In virtue of the generalized Plancherel's theorem, $\varphi(g)=\int \bar{g}(\chi)\,a(\chi)\,d\chi$, where $a(\chi)=|(f_W,\chi)|^2\in L_1(X)$. Take $U^*=\{|(a,g^*)-(a,0)|<\varphi(0)\}$. Then $U=\{|\varphi(g)-\varphi(0)|<\varphi(0)\}\subset W+(-W)\subset V$, for $\varphi(g)\equiv 0$ outside $W+(-W)$. Thus, it is proved, that the relative topology induced in G from G^* coincides with the original one in G.

2°. *The ring R' is regular: for every point $g_0^*\in G^*$ and its neighbourhood V^* there exists such a function $a(\chi)\in L_1(X)$ that $(a,g_0^*)\neq 0$ and $(a,g^*)\equiv 0$ outside V^*.* In fact, let W^* be the neighbourhood of the element g_0^* having a finite measure and such that $g_0^*+W^*+(-W^*)\subset V^*$. Put $\varphi(g^*)=\int f_{W^*}(g^*-g_0^*+h^*)\,f_{W^*}(h^*)\,dh^*$ (the integration passes over G^*). Evidently, $\varphi(g_0^*)>0$ and $\varphi(g^*)\equiv 0$ outside V^*. But, on the other hand, in virtue of the generalized Plancherel's theorem $\varphi(g^*)=(a,g^*)$, where $a(\chi)=\bar{g}_0^*(\chi)\left|\int f_{W^*}(h^*)\,\chi(h^*)\,dh^*\right|^2\in L_1(X)$. Note that $a(\chi)$ is a bounded function.

3°. *If $a(\chi)$ is a bounded function $\in L_1(X)$ and $(a,g)\equiv 0$ for all $g\in G$, then $a(\chi)\equiv 0$.* In fact, $a(\chi)\in L_2(X)$ and thus, owing to the generalized Plancherel's theorem, $a(\chi)=\int \chi(g)\,(a,\bar{g})\,dg\equiv 0$.

4°. It follows from 2° and 3°, that *G is everywhere dense in G^*.* In fact, if there exists a neighbourhood V^* of g_0^* non-intersecting with G, then in virtue of 2° there exists a bounded function $a(\chi)\in L_1(X)$ such that $(a,g)\equiv 0$ for all $g\in G$ and at the same time $(a,g_0^*)\neq 0$. But from 3° follows that $a(\chi)\equiv 0$. This completes the proof of Theorem 2. From it follows

The Pontrjagin's duality theorem. *If G is locally bicompact, then $G^*=G$.*

Commutative groups G, having an invariant measure and satisfying the second axiom of countability, for which $G\neq G^*$, may be characterized by

Theorem 3. *If $G\neq G^*$, every measurable subset of G having a finite positive measure is a non-measurable subset of G^* with the same exterior measure. Conversely, if G^* is locally bicompact and G is its non-measurable abstract subgroup, then, inducing the topology from G^* in G and defining the measure of Borel set $D\subset G$ as its exterior measure in G^*, we shall obtain a topological group with a totally additive invariant measure.*

On the whole we obtain, that *if on a commutative topological group G satisfying the second axiom of countability there exists a measure that satisfies the conditions mentioned in the beginning of the present note, then the group G is either locally compact, or an everywhere dense subgroup of a certain locally compact commutative group G^* with the second axiom of countability, such that the topology and measure in G are induced from G^*, whereby each measurable set of G with a finite positive measure is non-measurable in G^*.*

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MATHÉMATIQUES

ÉQUATIONS RÉGULIÈRES LINÉAIRES EN DIFFÉRENCES FINIES

Par M. HORNSTEIN

(Présenté par A. N. Kolmogoroff, de l'Académie, le 27. XII. 1940)

1. Une équation linéaire en différences finies (en abrégé «é. d. f.»)

$$D_{\nu+n}+a_1^{(\nu)}D_{\nu+n-1}+\ldots+a_n^{(\nu)}D_\nu=0 \quad (\nu=0, 1, 2 \ldots) \tag{A}$$

pour laquelle les limites $\lim a_i^{(\nu)}=a_i$ pour $\nu\to\infty$ existent, est nommée équation de Poincaré. Si l'équation caractéristique (en abrégé «é. c.»)

$$p^n+a_1p^{n-1}+\ldots+a_n=0 \tag{1}$$

possède des racines à modules différents

$$|p_1|>|p_2|>\ldots>|p_n|$$

alors, suivant le théorème de Poincaré (1), pour chaque intégrale D_ν de l'équation (A) il existe $\lim\frac{D_{\nu+1}}{D_\nu}$, et cette limite est égale à l'une des racines de l'é. c. (1). Et si l'é. c. possède des racines à modules égaux, il peuvent exister des intégrales pour lesquelles le rapport $\frac{D_{\nu+1}}{D_\nu}$ n'a pas de limite.

Convenons d'appeler la fonction D_ν *régulière*, s'il existe une limite finie $\lim\frac{D_{\nu+1}}{D_\nu}=p$, et de dire dans ce cas que «*la fonction* D_ν *appartient au nombre* p». Et si le rapport $\frac{D_{\nu+1}}{D_\nu}$ n'a pas de limite, la fonction D_ν sera nommée *irrégulière*. L'équation en différences finies (A) sera nommée *régulière*, si toutes ses intégrales sont régulières. Et si l'é. d. f. possède des intégrales irrégulières, nous la nommerons *irrégulière*. Si l'é. c. (1) possède des racines à modules égaux, mais qui ne sont pas égales, il y aura parmi les intégrales d'une telle équation des intégrales irrégulières. Perron (2) a démontré l'existence d'équations dont toutes les intégrales sont irrégulières. Mais on ne savait pas jusqu'à présent, s'il existe des équations *régulières* de ce type. Je vais montrer que de telles équations existent.

Considérons à cet effet le système de fonctions

$$D_\nu^{(1)}, D_\nu^{(2)}, \ldots, D_\nu^{(n)} \tag{2}$$

appartenant respectivement aux nombres p_i $(i=1, 2, \ldots, n)$ tels que $|p_1| \geqslant |p_2| \geqslant \ldots \geqslant |p_n|$, mais $p_i \neq p_k (i, k=1, 2, \ldots, n)$. Si $|p_i| > |p_k|$, il est facile à montrer que

$$\lim \frac{D_\nu^{(k)}}{D_\nu^{(i)}} = 0 \tag{3}$$

[la démonstration se trouve chez Perron (3)]. Mais si, par exemple,

$$|p_k| = |p_{k+1}| = \ldots = |p_{k+l-1}|$$

les fonctions correspondantes D_ν peuvent être choisies de telle manière que l'on ait

$$\lim \frac{D_\nu^{(k+s)}}{D_\nu^{(k+s-1)}} = 0 \quad (s=1, 2, \ldots, l-1). \tag{4}$$

Les fonctions (2) ainsi choisies seront linéairement indépendantes. Formons maintenant l'é. d. f. qui est vérifiée par le système d'intégrales (2). *Cette équation sera régulière.*

En effet, chaque intégrale de cette équation s'exprimera linéairement par les intégrales (2) dans la forme

$$D_\nu = C_1 D_\nu^{(1)} + C_2 D_\nu^{(2)} + \ldots + C_n D_\nu^{(n)}$$

d'où

$$\frac{D_{\nu+1}}{D_\nu} = \frac{D_{\nu+1}^{(1)}}{D_\nu^{(1)}} \cdot \frac{C_1 + C_2 \dfrac{D_{\nu+1}^{(2)}}{D_{\nu+1}^{(1)}} + \ldots + C_n \dfrac{D_{\nu+1}^{(n)}}{D_{\nu+1}^{(1)}}}{C_1 + C_2 \dfrac{D_\nu^{(2)}}{D_\nu^{(1)}} + \ldots + C_n \dfrac{D_\nu^{(n)}}{D_\nu^{(1)}}}.$$

En vertu de (3) et (4) tous les quotients $\dfrac{D_\nu^{(i)}}{D_\nu^{(1)}}$ $(i=2, \ldots, n)$ ont pour limite 0. Donc, $\lim \dfrac{D_{\nu+1}}{D_\nu} = \lim \dfrac{D_{\nu+1}^{(1)}}{D_\nu^{(1)}} = p_1$ pourvu que pour l'intégrale considérée on ait $C_1 \neq 0$. Et si $C_1 = 0$, $C_2 \neq 0$, nous aurons $\lim \dfrac{D_{\nu+1}}{D_\nu} = p_2$, etc.

Exemples. 1°. $D_\nu^{(1)} = 2^\nu (\nu+1)(\nu+2)$, $D_\nu^{(2)} = 2^\nu i^\nu$.
L'équation

$$D_{\nu+2} - 4\frac{\nu^2+5\nu+7}{(\nu+2)(\nu+3)-(\nu+1)(\nu+2)i} D_{\nu+1} +$$
$$+ 4i \frac{(\nu+3)(\nu+4)-(\nu+2)(\nu+3)i}{(\nu+2)(\nu+3)-(\nu+1)(\nu+2)i} D_\nu = 0$$

est régulière.

$$2^\circ.\ D_\nu^{(1)} = 2^\nu (\nu+1), \quad D_\nu^{(2)} = 2^\nu i^\nu (\nu+2).$$

L'équation

$$D_{\nu+2} - 4\frac{\nu^2+5\nu+5}{(\nu+2)^2-(\nu+1)(\nu+3)i} D_{\nu+1} +$$
$$+ 4i \frac{(\nu+3)^2-(\nu+2)(\nu+4)i}{(\nu+2)^2-(\nu+1)(\nu+3)i} D_\nu = 0$$

est irrégulière, parce que la limite de $\dfrac{D_\nu^{(2)}}{D_\nu^{(1)}}$ n'existe pas.

2. Montrons maintenant qu'il existe des équations d. f. régulières pour lesquelles les coefficients $a_i^{(\nu)}$ n'ont pas de limites. A cet effet déterminons les fonctions $D_\nu^{(1)}$ et $D_\nu^{(2)}$ de la manière suivante:

$$\left.\begin{aligned} D_{2\nu}^{(1)} &= (\nu+2)^2(\nu+3)\,a^{2\nu},\quad D_{2\nu+1}^{(1)} = (\nu+2)(\nu+3)^2\,a^{2\nu+1} \\ D_{2\nu}^{(2)} &= \frac{a^{2\nu}}{(\nu+1)^2},\quad D_{2\nu+1}^{(2)} = \frac{a^{2\nu+1}}{(\nu+1)(\nu+2)} \end{aligned}\right\} \tag{5}$$

où a est une constante. Ces fonctions vérifient les conditions

$$\lim \frac{D_{\nu+1}^{(1)}}{D_\nu^{(1)}} = a,\quad \lim \frac{D_{\nu+1}^{(2)}}{D_\nu^{(2)}} = a,\quad \lim \frac{D_\nu^{(2)}}{D_\nu^{(1)}} = 0. \tag{6}$$

Formons l'équation aux différences finies du second ordre

$$D_{\nu+2} + a_1^{(\nu)} D_{\nu+1} + a_2^{(\nu)} D_\nu = 0, \tag{7}$$

pour laquelle les fonctions (5) forment un système fondamental. Une intégrale quelconque de l'équation (7) s'exprime linéairement au moyen des intégrales $D_\nu^{(1)}$ et $D_\nu^{(2)}$ dans la forme

$$D_\nu = C_1 D_\nu^{(1)} + C_2 D_\nu^{(2)} \tag{8}$$

d'où en vertu de (6) nous aurons

$$\lim \frac{D_{\nu+1}}{D_\nu} = \lim \frac{D_{\nu+1}^{(1)}}{D_\nu^{(1)}} = a$$

pourvu que $C_1 \neq 0$; et si $C_1 = 0$, $C_2 \neq 0$, l'intégrale (8) se réduit à $D_\nu^{(2)}$, donc appartient aussi au nombre a. Ainsi l'équation (7) est régulière. D'autre part, le calcul des coefficients $a_{1,2}^{(\nu)}$ donne

$$a_1^{(2\nu)} = -\frac{5\nu+11}{2(\nu+2)}\,a,\quad a_1^{(2\nu+1)} = -\frac{5\nu+14}{3(\nu+3)}\,a,$$

$$a_2^{(2\nu)} = \frac{3}{2}a^2\frac{(\nu+1)(\nu+3)}{(\nu+2)^2},\quad a_2^{(2\nu+1)} = \frac{2}{3}a^2\frac{(\nu+1)(\nu+4)}{(\nu+2)(\nu+3)},$$

c'est-à-dire ils n'ont pas de limites.

Formons maintenant une équation aux différences finies du troisième ordre ayant pour intégrales les fonctions $D_\nu^{(1)}$, $D_\nu^{(2)}$, $D_\nu^{(3)}$ où $D_\nu^{(1)}$, $D_\nu^{(2)}$ sont définis par les égalités (5) et $D_\nu^{(3)} = b^\nu$, où b est une constante vérifiant la condition $|b| < |a|$. En vertu de la dernière condition on aura

$$\lim \frac{D_\nu^{(3)}}{D_\nu^{(2)}} = 0,\quad \lim \frac{D_\nu^{(3)}}{D_\nu^{(1)}} = 0.$$

L'équation obtenue sera régulière, mais ses coefficients, ainsi qu'on prouve par un calcul, n'ont pas de limites. On peut de même former une é. d. f. régulière d'ordre quelconque et qui n'est pas du type de Poincaré.

3. Supposons maintenant que l'é. d. f. (A) est régulière (du type de Poincaré ou non). Considérons un système fondamental d'intégrales (que nous allons nommer *normal*)

$$D_{\nu_0},\ D_{\nu_1},\ \ldots,\ D_{\nu(n-1)}. \tag{9}$$

et qui est défini par les conditions initiales

$$D_{ik} = \begin{cases} 1 & \text{pour } i = k \\ 0 & \text{pour } i \neq k \end{cases} \qquad (i, k = 0, 1, \ldots, n-1).$$

Chaque intégrale D_ν de l'équation (A), définie par les conditions initiales $D_0, D_1, \ldots, D_{n-1}$, s'exprime au moyen de la relation

$$D_\nu = D_0 D_{\nu_0} + D_1 D_{\nu_1} + \ldots + D_{n-1} D_{\nu(n-1)} \tag{10}$$

d'où il suit

$$\frac{D_{\nu+1}}{D_\nu} = \frac{D_{(\nu+1)(n-1)}}{D_{\nu(n-1)}} \times$$

$$\times \frac{D_0 \dfrac{D_{(\nu+1)0}}{D_{(\nu+1)(n-1)}} + D_1 \dfrac{D_{(\nu+1)1}}{D_{(\nu+1)(n-1)}} + \ldots + D_{n-2} \dfrac{D_{(\nu+1)(n-2)}}{D_{(\nu+1)(n-1)}} + D_{n-1}}{D_0 \dfrac{D_{\nu 0}}{D_{\nu(n-1)}} + D_1 \dfrac{D_{\nu 1}}{D_{\nu(n-1)}} + \ldots + D_{n-2} \dfrac{D_{\nu(n-2)}}{D_{\nu(n-1)}} + D_{n-1}}.$$

En vertu de la régularité de l'équation (A) tous les quotients du second membre de cette dernière égalité ont des limites et d'ailleurs ces limites

$$\lim \frac{D_{\nu i}}{D_{\nu(n-1)}} = \alpha_i \quad (i = 0, 1, \ldots, n-2) \tag{11}$$

peuvent être égales à un nombre fini $\neq 0$, à 0, ou bien à ∞. Supposons d'abord qu'elles sont finies. Alors en posant $\lim \frac{D_{(\nu+1)(n-1)}}{D_{\nu(n-1)}} = p_1$, nous aurons

$$\lim \frac{D_{\nu+1}}{D_\nu} = p_1 \tag{12}$$

pourvu que les conditions initiales soient choisies de telle manière que l'on ait

$$\alpha_0 D_0 + \alpha_1 D_1 + \ldots + \alpha_{n-2} D_{n-2} + D_{n-1} \neq 0. \tag{13}$$

Ainsi toutes les intégrales de l'équation (A), dont les conditions initiales vérifient la relation (13), appartiennent à un même nombre p_1. Je les nommerai *intégrales de la première classe* de l'é. d. f. (A).

Choisissons maintenant les conditions initiales $D_0, D_1, \ldots, D_{n-1}$ de telle manière que l'on ait l'égalité

$$\alpha_0 D_0 + \alpha_1 D_1 + \ldots + \alpha_{n-2} D_{n-2} + D_{n-1} = 0. \tag{14}$$

En déterminant de cette équation D_{n-1} et en le substituant dans (10), nous aurons

$$D_\nu = D_0 D'_{\nu 0} + D_1 D'_{\nu 1} + \ldots + D_{n-2} D'_{\nu(n-2)} \tag{15}$$

où l'on a introduit les notations

$$D'_{\nu i} = D_{\nu i} - \alpha_i D_{\nu(n-1)} \quad (i = 1, 2, \ldots, n-2).$$

Les valeurs $D'_{\nu i}$ comme combinaisons linéaires des intégrales de l'é. d. f. (A) sont aussi des intégrales de cette équation.

On trouve de (15) comme précédemment que toutes les intégrales dont les conditions initiales vérifient non seulement (14), mais encore la relation

$$\alpha'_0 D_0 + \alpha'_1 D_1 + \ldots + \alpha'_{n-3} D_{n-3} + D_{n-2} \neq 0 \tag{16}$$

où

$$\alpha'_i = \lim \frac{D'_{\nu i}}{D_{\nu(n-2)}} \quad (i = 0, 1, \ldots, n-3)$$

(d'ailleurs tous les α'_i sont supposés finis), appartiennent au même nombre p_2, de sorte que pour elles on a

$$\lim \frac{D_{\nu+1}}{D_\nu} = p_2. \tag{17}$$

Ces intégrales seront nommées *intégrales de seconde classe* de l'é. d. f. (A).

Si $D_\nu^{(1)}$ et $D_\nu^{(2)}$ désignent respectivement les intégrales de première et de seconde classe, alors, en partant de ce que les conditions initiales

pour $D_\nu^{(1)}$ vérifient la relation (13) et pour $D_\nu^{(2)}$ la relation (14), il est facile d'obtenir

$$\lim \frac{D_\nu^{(2)}}{D_\nu^{(1)}} = 0 \tag{18}$$

et ensuite, en tenant compte de ce que pour $D_\nu^{(1)}$ la relation (12) a lieu et pour $D_\nu^{(2)}$ on a (17) nous obtenons

$$|p_1| \geqslant |p_2|.$$

Ce procédé peut être prolongé. Ainsi on obtient le

Théorème. *Si* (A) *est une é. d. f. régulière, toutes ces intégrales se divisent en n classes telles que toutes les intégrales de la k-ième classe* $(k = 1, 2, \ldots, n)$ *appartiennent à un même nombre* p_k. *Si* $D_\nu^{(r)}$ *et* $D_\nu^{(s)}$ *désignent deux intégrales quelconques de classes r et s respectivement, on a la relation*

$$\lim \frac{D_\nu^{(s)}}{D_\nu^{(r)}} = 0 \quad \textit{pour} \quad s > r. \tag{19}$$

Il en résulte que les nombres p_i *vérifient les conditions*

$$|p_1| \geqslant |p_2| \geqslant \ldots \geqslant |p_n|. \tag{20}$$

D'ailleurs, les intégrales de la k-ième classe $(1 \leqslant k \leqslant n)$ *sont définies par les conditions initiales, vérifiant les relations*

$$\alpha_0^{(s)} D_0 + \alpha_1^{(s)} D_1 + \ldots + \alpha_{n-s-2}^{(s)} D_{n-s-2} + D_{n-s-1} = 0 \quad (s = 0, 1, \ldots, k-2),$$
$$\alpha_0^{(k-1)} D_0 + \alpha_1^{(k-1)} D_1 + \ldots + \alpha_{n-k-1}^{(k-1)} D_{n-k-1} + D_{n-k} \neq 0$$

(on prend $\alpha_i^{(0)}$ égal à α_i), où

$$\left.\begin{aligned} \alpha_i^{(s)} &= \lim \frac{D_{\nu i}^{(s)}}{D_{\nu\,(n-s-1)}} \quad (i = 0, 1, \ldots, n-s-2), \\ D_{\nu i}^{(s)} &= D_{\nu i}^{(s-1)} - \alpha_i^{(s-1)} D_{\nu\,(n-s)}^{(s-1)} \quad (i = 0, 1, \ldots, n-s-1) \end{aligned}\right\} \tag{21}$$

(on prend $D_{\nu i}^{(0)}$ égal à $D_{\nu i}$).

Remarque: 1) Il peut arriver que les nombres (11) ne sont pas tous finis. Mais parmi les intégrales (9) du système normal il existe au moins une qui appartient au nombre à module maximal, et s'il existe plusieurs intégrales appartenant à ce nombre, il y en a parmi elles une telle que le rapport de toutes les autres à cette dernière est un nombre fini. Désignons cette dernière par $D_{\nu k}$. Alors, au lieu de (11) posons

$$\lim \frac{D_{\nu i}}{D_{\nu k}} = \alpha_i \quad (i = 0, 1, \ldots, k-1, k+1, \ldots, n).$$

On fera le même par les autres α à indices supérieurs.

2) Si l'équation (A) est du type de Poincaré, les nombres (18) seront les racines de l'é. c. En particulier, si les racines de l'é. c. ont des modules différents, en vertu du théorème de Poincaré [1], l'é. d. f. sera régulière. Mais alors, d'après le théorème précédent, il existe un système fondamental formé de n intégrales appartenant respectivement aux racines de l'é. c. Ainsi on obtient très simplement le théorème de Perron [3].

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LITTÉRATURE CITÉE

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MATHÉMATIQUES

SUR L'UNIFORMISATION DES ENSEMBLES *A*

Par W. JANKOFF

(Présenté par S. L. Soboleff, de l'Académie, le 2. III. 1940)

Ainsi qu'il a été démontré par M. N. Lusin, un ensemble A peut être uniformisé au moyen d'un ensemble qui appartient au corps B construit sur les ensembles A et CA (1).

Dans la présente Note nous obtenons le même résultat par une autre méthode, et nous étudions les propriétés descriptives de l'ensemble uniformisant. Nous suivons la terminologie proposée par M. N. Lusin (2).

Un ensemble plan qui possède un point au plus sur chaque parallèle à l'axe OY sera nommé une courbe. Nous dirons que la courbe H est mesurable (ou bien possède la propriété de Baire), si la fonction $y=f(x)$ définie sur la projection de H sur l'axe OX et telle que tous les points à coordonnées $(x, f(x))$ appartiennent à H, est mesurable ou bien possède la propriété de Baire. Nous dirons que la fonction $f(x)$ représente H. Soit $\mathcal{E}$ un ensemble plan élémentaire G_δ et $H-$l'ensemble de ses points inférieurs. Alors H est une courbe mesurable et possédant la propriété de Baire Cela résulte immédiatement de ce que tous les ensembles de Lebesgue de la fonction qui répresente H sont des différences de deux ensembles A (c'est-à-dire des ensembles A_ρ), donc ils sont mesurables et possèdent la propriété de Baire.

Théorème. *Chaque ensemble A plan est uniformisable au moyen d'une courbe possédant la propriété de Baire et qui est un ensemble $A_{\rho\sigma\delta}$.*

Démonstration. Soit $\mathcal{E}$ un ensemble A plan situé dans le plan OXY. Désignons par E un ensemble élémentaire G_δ situé dans l'espace $OXYZ$ et ayant pour projection $\mathcal{E}$. On a

$$E=\prod_n \sum_k \delta_{nk} \qquad (1)$$

où tous les δ_{nk} sont des parallélepipèdes ouverts. Désignons par H la projection de E sur OXZ. L'ensemble H est encore un G_δ élémentaire et d'ailleurs

$$H=\prod_n \sum_k \pi_{xz}\delta_{nk}.$$

Désignons par K l'ensemble des points de H inférieurs par rapport à l'axe OZ. Soit U l'ensemble de tous les points situés sur les parallèles à l'axe OY et passant par les points de l'ensemble K. Posons $V=UE$ et $\Gamma=\pi_{xy}V$.

Montrons que l'ensemble Γ vérifie toutes les conditions du théorème, c'est-à-dire:

1) Γ uniformise E;

2) Γ est un $A_{\rho\sigma\delta}$;

3) la courbe Γ est mesurable et possède la propriété de Baire.

1. Comme E est un G_δ élémentaire, sur chaque droite passant par K et parallèle à l'axe OY il existe précisément un point de l'ensemble V. D'ailleurs, comme H est un G_δ élémentaire, la projection de K sur OX coïncide avec la projection de H, donc avec celle de E sur OX.

Il en résulte que Γ uniformise en effet E.

2. Démontrons maintenant que les ensembles de Lebesgue de la fonction représentant Γ sont des $A_{\rho\sigma}$. En effet, chaque ensemble de Lebesgue de la fonction représentant Γ est une somme d'une infinité dénombrable d'ensembles de Lebesgue pour la fonction qui représente K. Pour le voir, on peut raisonner de la manière suivante. Soit $\Gamma_{y_1y_2}$ la partie de l'ensemble Γ située exactement entre les droites $y=y_1$ et $y=y_2$. Soient

$$\delta^1, \delta^2, \ldots, \delta^n, \ldots$$

tous les rectangles δ_{nk} de (1) situés entièrement entre les plans $y=y_1$ et $y=y_2$. Posons

$$V_n = V\delta^n$$

et désignons

$$\Gamma_n = \pi_{xy} V_n; \quad K_n = \pi_{xz} V_n.$$

Il est alors évident que $\pi_x K_n$ est un certain ensemble de Lebesgue de la fonction représentant K et

$$\Gamma_{y_1y_2} = \sum_n \Gamma_n; \quad \pi_x \Gamma_n = \pi_x V_n$$

donc

$$\pi_x \Gamma_{y_1y_2} = \sum_n \pi_x V_n.$$

Il en résulte que la courbe Γ est un $A_{\rho\sigma\delta}$, parce que Γ peut être représentée dans la forme

$$\Gamma = \prod_n \sum_k \tau_{nk} x \Gamma_{nk}$$

où les Γ_{nk} sont des ensembles de Lebesgue et τ_{nk} — certains intervalles.

3. Comme les ensembles $A_{\rho\sigma}$ sont mesurables et possèdent la propriété de Baire, la courbe Γ est aussi mesurable et possède la propriété de Baire.

Remarque. On peut démontrer par la même méthode qu'un ensemble plan mesurable B peut être uniformisé par une courbe CA mesurable et possédant la propriété de Baire. Dans ce cas l'ensemble E peut être considéré comme uniforme par rapport aux parallèles à l'axe OZ.

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MÉCANIQUE

QUELQUES MÉTHODES D'APPROXIMATION POUR LA RÉSOLUTION DU PROBLÈME DE FILTRATION DANS LE MILIEU À DEUX COUCHES

Par N. KALININE

(Présenté par N. E. Kotchine, de l'Académie, le 3. XII. 1940)

1. La méthode de résoudre le problème du mouvement plan du flux de filtration sous une digue à fondement rectiligne ou bien sous une rainure verticale dans un sol, composé d'un système de couches horizontales de perméabilité différente, proposée par Riesenkampf (1), consiste dans la substitution du fondement d'un bâtiment hydrotechnique par un système de tourbillons. De là on trouve l'intensité de ces tourbillons au moyen d'une équation obtenue en tenant compte des conditions limites. Il est clair, que cette méthode s'étend facilement au cas, où le fondement du bâtiment hydrotechnique est d'une configuration arbitraire.

Supposons que le flux de filtration se déplace sous une digue, dont le fondement présente une courbe C, dans un milieu composé de deux couches horizontales de l'épaisseur h_1 et h_2 et de perméabilité $\varkappa_1$ et $\varkappa_2$. Dirigeons l'axe x suivant la frontière du bassin de l'eau, l'axe vertical y de bas en haut. L'origine des coordonnées doit être placée de façon que les coordonnées des points de rencontre de la courbe C avec l'axe des x soient égales à $-a$ et $+a$.

Comme la composante de la vitesse de filtration dans la direction horizontale sur l'axe des x est nulle ($u=0$), le flux considéré peut être prolongé, au point de vue analytique, dans le demi-plan supérieur. Alors nous obtiendrons un mouvement de circulation du liquide dans un sol à trois couches autour du contour $C+C'$, où C' est une courbe symétrique de C par rapport à l'axe des x. Il est évident qu'un tel flux peut être obtenu en distribuant les tourbillons d'intensité variable suivant le contour $C+C'$ et en choisissant cette intensité de façon que soient vérifiées les conditions limites sur les surfaces solides, de même que sur les lignes de séparation des couches.

Avant de passer à la résolution de ce problème, nous allons trouver dans la région indiquée le flux de deux tourbillons de l'intensité unitaire, situés dans les points symétriques par rapport à l'axe des x. Nous supposerons, afin de simplifier le problème, que les tourbillons soient situés dans une couche au coefficient de filtration $\varkappa_1$. D'après la méthode de Risenkampf(1), on obtiendra la solution cherchée sous la forme des intégrales de Fourier:

$$u_1 - iv_1 = \frac{1}{2\pi i}\left[\frac{1}{z-\zeta}+\frac{1}{z-\bar{\zeta}}+4\int_0^\infty \frac{(\varkappa_2 \operatorname{sh} h_2 u - \varkappa_1 \operatorname{ch} h_2 u)\operatorname{ch}\eta_1 u e^{-h_1 u}\sin u\,(z-\xi)}{(\varkappa_2+\varkappa_1)\operatorname{ch} hu+(\varkappa_1-\varkappa_2)\operatorname{ch}(h_1-h_2)u}\,du\right]$$

$$u_2 - iv_2 = \frac{2\varkappa_2}{\pi}\int_0^\infty \frac{\operatorname{ch}\eta_1 u \cos u\,(z-\xi+ih)}{(\varkappa_2+\varkappa_1)\operatorname{ch} hu+(\varkappa_1-\varkappa_2)\operatorname{ch}(h_1-h_2)u}\,du.$$

La première de ces formules définit la vitesse complexe dans une couche avec le coefficient de filtration $\varkappa_1$, la seconde — dans une couche

au coefficient de filtration $\varkappa_2$. L'expression $\zeta = \xi + i\eta$ correspond au point, où est situé le tourbillon $h = h_1 + h_2$. Sur la frontière des couches, pour $y = -h_1$, sont satisfaites les conditions $v_1 = v_2$; $\frac{u_1}{\varkappa_1} = \frac{u_2}{\varkappa_2}$; de même, sur la surface non perméable (pour $y = -h$) $v_2 = 0$.

Après avoir distribué les tourbillons de l'intensité $\nu(s)\,dS$ le long du contour C, on trouvera les vitesses complexes de filtration au-dessous d'une digue dans un sol à deux couches sous une forme:

$$U_1 - iV_1 = \int_C \nu(s)\,(u_1 - iv_1)\,dS; \quad U_2 - iV_2 = \int_C \nu(s)\,(u_2 - iv_2)\,dS.$$

En approchant le point z au point σ du contour C, on trouvera, à la condition que la composante normale de la vitesse disparaisse, une équation intégrale définissant la fonction $\nu(s)$: v. p. $\int_C \nu(s)\,K(\sigma, s)\,dS = 0$

où v. p. désigne la valeur fondamentale dans le sens de Cauchy. La fonction $\nu(s)$ doit satisfaire de plus à la condition

$$\int \nu(s)\,dS = 2\varkappa_1 H \tag{1}$$

où H est la différence des niveaux dans les biefs supérieur et inférieur.

2. Dans le cas du déversoir plan, représenté par le segment $(-a, +a)$ de l'axe x, on aura l'équation intégrale suivante:

$$\text{v. p.} \int_{-a}^{a} \nu(\xi)\left[\frac{1}{\xi - x} + 2\int_0^\infty \frac{e^{-h_1 u}(\varkappa_2 \operatorname{sh} h_2 u - \varkappa_1 \operatorname{ch} h_2 u)\sin(\xi - x)\,u}{(\varkappa_2 + \varkappa_1)\operatorname{ch} hu - (\varkappa_2 - \varkappa_1)\operatorname{ch}(h_1 - h_2)\,u}\,du\right]d\xi = 0. \tag{2}$$

Si la couche inférieure s'étend indéfiniment dans la profondeur ($h_2 = \infty$), l'équation (2) se présentera de la façon suivante:

$$\text{v. p.} \int_{-a}^{a} \nu(\xi)\left[\frac{1}{\xi - x} + 2\lambda\int_0^\infty \frac{e^{-2h_1 u}\sin(\xi - x)\,u}{1 - \lambda e^{-2h_1 u}}\,du\right]d\xi = 0 \tag{3}$$

ou bien

$$\text{v. p.} \int_{-a}^{a} \nu(\xi)\left[\frac{1}{\xi - x} + 2(\xi - x)\sum_{n=1}^{\infty}\frac{\lambda^n}{(\xi - x)^2 + 4n^2h_1^2}\right]d\xi = 0 \tag{4}$$

où
$$\lambda = \frac{\varkappa_2 - \varkappa_1}{\varkappa_2 + \varkappa_1}. \tag{5}$$

Il est facile à réduire l'équation (4) à l'équation intégrale non homogène de Fredholm de 2-ème espèce. Ecrivons-la sous la forme

$$\text{v. p.} \int_{-a}^{a} \frac{\nu(\xi)\,d\xi}{\xi - x} = \psi(x) \tag{6}$$

où
$$\psi(x) = -2\int_{-a}^{a}(\xi - x)\sum_{n+1}^{\infty}\frac{\lambda^n}{(\xi - x)^2 + 4n^2h_1^2}\cdot\nu(\xi)\,d\xi.$$

Servons nous de la formule d'inversion de l'équation intégrale (6)

$$\nu(x) = -\frac{1}{\pi^2\sqrt{a^2 - x^2}}\,\text{v. p.}\int_{-a}^{a}\frac{\psi(s)\sqrt{a^2 - s^2}}{s - x}\,ds + \frac{C}{\pi\sqrt{a^2 - x^2}}$$

où $C = \int_{-a}^{a}\nu(s)\,dS$, donnée dans l'article de Keldych et Lavrentieff (²) et que l'on trouve au moyen des formules proposées par Sedoff (³) dans la théo-

rie d'une aile fine. Après des calculs simples nous obtiendrons une équation intégrale suivante, équivalente à l'équation (4):

$$\nu(x) = \int_{-a}^{a} K(x, \xi)\, \nu(\xi)\, d\xi + f(x) \qquad (7)$$

où

$$K(x, \xi) = -\frac{\sqrt{2}}{\pi\sqrt{a^2 - x^2}} \sum_{n=1}^{\infty} \lambda^n \frac{(\xi - x) C_n + 2nh_1 D_n}{(\xi - x)^2 + 4n^2 h_1^2},$$

$$C_n = \sqrt{\sqrt{(\xi^2 - 4n^2h_1^2 - a^2)^2 + 16n^2\xi^2 h_1^2} + \xi^2 - 4n^2h_1^2 - a^2}$$

$$D_n = \sqrt{\sqrt{(\xi^2 - 4n^2h_1^2 - a^2)^2 + 16n^2\xi^2 h_1^2} - \xi^2 + 4n^2h_1^2 + a^2}$$

$$f(x) = \frac{2\varkappa_1 H (1 + \lambda)}{\pi (1 + \lambda)} \cdot \frac{1}{\sqrt{a^2 - x^2}}.$$

3. Indiquons quelques méthodes approximatives de résolution de l'équation (7) pour les petites valeurs de quelques paramètres.

Supposons, par exemple, que le paramètre λ ait la valeur voisine de zero (les coefficients de filtration $\varkappa_1$ et $\varkappa_2$ se distinguent peu l'un de l'autre). Alors la fonction inconnue $\nu(x)$ peut être représentée sous la forme d'une série $\nu(x) = \nu_0(x) + \lambda\nu_1(x) + \lambda^2\nu_2(x) + \ldots$ Soient de même

$$K(x, \xi) = \lambda K_1(x, \xi) + \lambda^2 K_2(x, \xi) + \ldots$$
$$f(x) = f_0(x) + \lambda f_1(x) + \lambda^2 f_2(x) + \ldots$$

En substituant ces trois séries dans l'équation (7), et en égalisant les coefficients des termes à la même puissance de λ, l'on aura

$$\nu_0(x) = \frac{2\varkappa_1 H}{\pi\sqrt{a^2 - x^2}}; \quad \nu_1(x) = -\frac{2\sqrt{2}H\varkappa_1}{\pi^2\sqrt{a^2 - x^2}} \int_{-a}^{a} \frac{(\xi - x) C_1 + 2h_1 D_1}{(\xi - x)^2 + 4h_1^2} \cdot \frac{d\xi}{\sqrt{a^2 - \xi^2}}$$

où

$$C_1 = \sqrt{\sqrt{(\xi^2 - 4h_1^2 - a^2) + 16\xi h_1^2} + \xi^2 - 4h_1^2 - a^2}$$

$$D_1 = \sqrt{\sqrt{(\xi^2 - 4h_1^2 - a^2)^2 + 16h_1^2\xi^2} - \xi^2 + 4h_1^2 + a^2}$$

et, enfin, sous la forme générale

$$\nu_n(x) = \int_{-a}^{a} (\nu_0 K_n + \nu_1 K_{n-1} + \ldots + \nu_{n-1} K_1)\, d\xi + f_n(x).$$

Soit, d'autre part, λ voisin de -1 (le coefficient de filtration $\varkappa_2$ est voisin de zéro). $\lambda = -1 + \mu$, où $\mu = \frac{2\varkappa_2}{\varkappa_2 + \varkappa_1}$. Transformons (3); l'on a

$$\text{v. p.} \int_{-a}^{a} \nu(\xi) \left[\frac{1}{\operatorname{sh} \frac{\pi(\xi - x)}{2h_1}} + \sum_{n=1}^{\infty} \mu^n A_n(x, \xi) \right] d\xi = 0 \qquad (8)$$

où

$$A_n(x, \xi) = \frac{4h_1}{\pi} \int_0^{\infty} \frac{e^{-2nh_1 u} \sin(\xi - x)\, u}{(1 + e^{-2h_1 u})^{n+1}}\, du.$$

L'équation (8) peut aussi être réduite à l'équation intégrale non homogène de Fredholm de 2-ème espèce. Pour cela écrivons la de la façon suivante:

$$\text{v. p.} \int_{-a}^{a} \frac{\nu(\xi)\, d\xi}{\operatorname{sh} \frac{\pi(\xi - x)}{2h_1}} = \psi(x) \qquad (9)$$

où

$$\psi(x) = -\sum_{n=1}^{\infty} \mu^n \int_{-a}^{a} A_n(x, \xi)\, \nu(\xi)\, d\xi.$$

Il est facile d'obtenir la solution de l'équation intégrale (8) au moyen des formules obtenues par Sedoff [3] dans la théorie des réseaux. L'on a

$$\nu(x) = -\frac{\operatorname{ch}\frac{\pi x}{2h_1}}{4h_1^2\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}} \int_{-a}^{a} \frac{\psi(s)\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}}{\operatorname{sh}\frac{\pi(s-x)}{2h_1} \cdot \operatorname{ch}\frac{\pi s}{2h_1}}\, dS + \\ + \frac{C}{\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}} \qquad (10)$$

où la constante C peut être obtenue de la condition (1). En substituant dans (10) la valeur de $\psi(s)$, on trouvera une équation intégrale de Fredholm de 2-ème espèce:

$$\nu(x) = \int_{-a}^{a} \nu(\xi)\, K(x, \xi)\, d\xi + f(x) \qquad (11)$$

où

$$K(x, \xi) = \frac{\operatorname{ch}\frac{\pi x}{2h_1}}{4h_1^2\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}} \sum_{n=1}^{\infty} \mu^n \int_{-a}^{a} \frac{A_n(s, \xi)\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}}{\operatorname{sh}\frac{\pi(s-x)}{2h_1} \cdot \operatorname{ch}\frac{\pi s}{2h_1}}\, ds$$

$$f(x) = \frac{C}{\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}}.$$

Pour μ petit, on peut trouver la fonction inconnue $\nu(x)$ sous la forme d'une série $\nu(x) = \nu_0(x) + \mu\nu_1(x) + \mu^2\nu_2(x) + \ldots$

Substituons cette série dans (11). Après avoir égalisé les coefficients des termes à la même puissance de μ, on trouvera, en particulier:

$$\nu_0(x) = \frac{\varkappa_1 \pi H \operatorname{ch}\frac{\pi a}{2h_1}}{2hK} \cdot \frac{1}{\sqrt{\operatorname{sh}^2\frac{\pi a}{2h_1} - \operatorname{sh}^2\frac{\pi x}{2h_1}}}$$

où

$$K = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - \varkappa^2 \sin^2\varphi}}, \qquad \varkappa = \operatorname{th}\frac{\pi a}{2h_1}.$$

La méthode des autres approximations est évidente.

4. De la même façon on peut considérer la question d'une rainure verticale enfoncée, ainsi que d'autres exemples.

L'auteur tient à exprimer sa reconnaissance à P. J. Kotchine pour de précieux conseils et à B. K. Risenkampf pour les observations dont l'auteur s'était servi.

Chaire de mécanique.
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PHYSICS

THE THEORY OF HYSTERESIS OF POLYCRYSTALLINE FERROMAGNETICS IN WEAK MAGNETIC FIELDS

By E. KONDORSKY

(Communicated by P. L. Kapitza, Member of the Academy, 18. XII. 1940)

In a previous communication (1) an attempt has been made to work out on the basis of modern conceptions of the phenomenon of magnetization (2) an approximate theory of hysteresis for those cases, in which the mutual magnetic action of the separate parts of the body may be neglected. In the present paper there will be discussed the possibility of developing an approximate theory of magnetic properties of polycrystalline ferromagnetics, account being taken of the magnetic interaction between the separate parts.

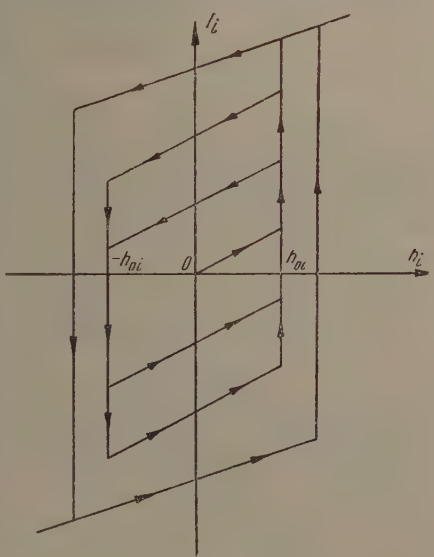

Fig. 1.

Let us consider a polycrystalline ferromagnetic, consisting of separate parts, with a definite direction of easy magnetization in each of them. Such parts can be in some cases single grains of the polycrystalline ferromagnetic (i. e. the crystallites) or groups of grains, in other cases parts of grains. On the basis of experimental results obtained from the investigation of ferromagnetics with one axis of easy magnetization (3), we shall suppose that each such part has magnetic characteristics, similar to those shown in Fig. 1, in particular that each part possesses an almost parallel-sided hysteresis loop. Let us assume, furthermore, that in these parts, as in larger ferromagnetics with an almost parallel-sided hysteresis loop, a number of values of residual magnetism may be obtained instead of one.

Let I denote the mean value of magnetization; H, the intensity of the field, which is applied to the ferromagnetic; χ_r, the reversible susceptibility; I_i, the mean value of the longitudinal component of magnetization of the part i; h_i, field intensity within this part. Moreover, let us suppose that the ferromagnetic is on the average isotropic, at least in the plane perpendicular to the direction H, i. e. the normal component of I equals zero.

In this case we may suppose as the first approximation that the direction of the local field h_i coincides with the direction H. To simplify calculations, let us confine ourselves to the case, when the value of reversible susceptibility of the separate parts may be considered equal* to the reversible susceptibility χ_r of the ferromagnetic as a whole. In so far as the values of χ_r are considered equal in all the parts, the ferromagnetic may be looked upon, from the formal point of view, as a result of superposition of the two media; one possessing a hysteresis and a zero reversible susceptibility, and another—a medium without hysteresis, but with an χ_r susceptibility.

The mean value of magnetization of the ferromagnetic may be presented therefore as the sum of the values of magnetization of the first and second medium, namely:

$$I = \sum_i (I_i - \chi_r h_i)\, v_i + \chi_r H. \tag{1}$$

Here v_i is the concentration of parts per unit volume. The field h_i within the part i may be presented as the sum of the external field H and the magnetic field produced by all the parts of the first medium:

$$h_i = H + N_i[(I - \chi_r H) - (I_i - \chi_r h)]. \tag{2}$$

Here N_i is a coefficient depending upon the shape of the part i and on the reversible susceptibility μ_r of the second medium

$$N_i = \frac{N_{i0}}{\mu_r} = \frac{N_{i0}}{1 + 4\pi\chi_r} \tag{3}$$

where N_{i0} is the demagnetization factor of part i.

Let us denote

$$I_0 = I - \chi_r H \quad (4) \qquad I_{0i} = I_i - \chi_r h_i. \tag{5}$$

Evidently, when $\chi_r = 0$, $I_0 = I$ and $I_{0i} = I_i$. Thus I_0 represents the average magnetization of the first medium, I_{0i} — the longitudinal component of magnetization of its i-th part. Taking (4) and (5) into account, we obtain from (1) and (2):

$$I_0 = \sum_i I_{0i}\, v_i \tag{6}$$

$$I_{0i} = I_0 + \frac{1}{N_i}(H - h_i). \tag{7}$$

Let us denote by h_{0i} the critical value of the field intensity h_i, at which there occurs in the part i an irreversible change in magnetization. All the parts may be divided into three groups:
1) parts, in which $h_i < h_{0i}$; 2) those, in which $h_i = h_{0i}$, and lastly, 3) parts, in which $h_i > h_{0i}$. The concentration of parts per unit volume we shall respectively denote by $v_i^{(1)}$, $v_i^{(2)}$ and $v_i^{(3)}$.

In accordance with our initial supposition, both the magnetization curves and the hysteresis loops of the considered parts of the ferromagnetic are parallel-sided. Since from (4) it is evident that I_{0i} represents the longitudinal component of magnetization in the part i, in the case of the reversible susceptibility equalling zero, it follows, that the values of I_{0i} for the parts of the three groups are:

$$\left.\begin{array}{l} \text{for the first group } (h_i < h_{0i})\ I_{0i} = 0 \\ \text{for the second group } (h_i = h_{0i})\ 0 \leqslant I_{0i} \leqslant I_s \cos\vartheta_i \\ \text{for the third group } (h_i > h_{0i})\ I_{0i} = I_s \cos\vartheta_i \end{array}\right\} \tag{8}$$

* At the expense of complicating the calculations, it can be shown that the difference in the value of reversible susceptibility of the separate parts is essential only within the range of high values of H, at $H \geqslant H_L$ (see below).

where I_s is the magnetization of saturation; ϑ_i, the angle formed by the axis of easy magnetization of the part i and the direction of the field H. According to (7), for parts of the second group*:

$$I_{0i} = I_0 + \frac{1}{N_i}(H - h_{0i}) \geqslant 0. \tag{8'}$$

Taking into account (8) and (8'), we obtain from (5'):

$$I_0 = \sum_i \left[I_0 + \frac{1}{N_i}\left(H - h_{0i}\right) \right] v_i^{(2)} + \sum_i I_s \cos\vartheta_i v_i^{(3)}. \tag{9}$$

The parts of the third group make their first appearance when the field intensity attains a certain definite value.

In the present paper the discussion will be devoted to the case when the maximum field intensity H_m does not exceed the limiting value of H_L. In this instance the parts of the third group are absent ($v_i^{(3)} = 0$) and the maximum value of I_{0m}, corresponding to the maximum value of the field intensity H_m is, in accordance with (9), equal to

$$I_{0m} = \sum \left[I_{0m} + \frac{1}{N_i}\left(H - h_{0i}\right) \right] v_i^{(2)} = \sum (I_{0i})_m \; v_i^{(2)}. \tag{10}$$

Here

$$(I_{0i})_m = I_{0m} + \frac{1}{N_i}(H - h_{0i}) \tag{11}$$

represents the value of I_{0i} at $H = H_m$. The limits of the sum in (10) depend upon the values of H_m and I_{0m}. Namely, the summation (10) must be extended over all the parts, the values h_{0i} and N_i of which are bound by the inequality (9) or its equivalent expression:

$$h_{0i} \leqslant H_m + N_i I_{0m}. \tag{12}$$

Let I', I'_0, I'_i, I'_{0i} and v'_i respectively represent the values of I, I_0, I_i, I_0 and v_i in the course of magnetic field reversal, i. e. at $-H_m < H < H_m$**.

In examining the process of reversal, it is convenient to divide the parts of the ferromagnetic, in which $(I_{0i})_m > 0$, into the following subgroups: 1) parts, in which $h_i > -h_{0i}$; 2) those, in which $h_i = -h_{0i}$, and, lastly, 3) parts, in which $h_i < -h_{0i}$. The concentration of parts per unit volume we shall denote for the three subgroups: $v_i'^{(1)}$, $v_i'^{(2)}$ and $v_i'^{(3)}$. Since in the general case the parts, in which $(I_{0i})_m > 0$ at $H = H_m$, belonged either to the second or to the third group, the following relation holds true:

$$\sum_i (v_i'^{(1)} + v_i'^{(2)} + v_i'^{(3)}) = \sum_i (v_i^{(2)} + v_i^{(3)})_{H = H_m}. \tag{13}$$

It will be remembered that in accordance with our initial supposition the parts of the ferromagnetic possess a parallel-sided hysteresis loop. Consequently, the values of I'_{0i} will equal:

$$\left.\begin{array}{ll} \text{for the 1 subgroup } (h_i > -h_{0i}) & I'_{0i} = (I_{0i})_m \\ \text{for the 2 subgroup } (h_i = -h_{0i}) & -I_s \cos\vartheta_i \leqslant I'_{0i} \leqslant (I_{0i})_m \\ \text{for the 3 subgroup } (h_i < -h_{0i}) & I'_{0i} = -I_s \cos\vartheta_i \end{array}\right\} \tag{14}$$

In agreement with (7) and (14) for parts of the 2 subgroup

$$I'_{0i} = I'_0 + \frac{1}{N_i}(H + h_{0i}) \leqslant (I_{0i})_m. \tag{15}$$

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* Inequalities (8') hold true also for parts of the third group.

** If (I_{0i}) would approximately have equalled $I_s \cos\vartheta_i$, the increase of the oppositely magnetized phase in the first moment would have been accompanied by a growth of the surface energy of the phases and would always have taken place at $h_i < -h_{0i}$ [4]. However, if $(I_{0i})_m$ is sufficiently distinct from $I_s \cos\vartheta_i$, the conditions of this increase of the oppositely magnetized phase in no way differ from conditions of the growth of the phase magnetized positively, and the growth of the formed must begin at $h_i = -h_{0i}$.

If $H_m < H_L$, then at $-H_m < H < H_m$, there are present in the ferromagnetic only parts of the 2 first subgroups and the first group. Taking into consideration and substituting the value of I'_{0i} from (14) and (15) into (5) we obtain:

$$I'_0 = \sum_i (I_{0i})_m v_i'^{(1)} + \sum_i [I'_0 + \frac{1}{N_i}(H + h_{0i})] v_i'^{(2)}. \qquad (16)$$

This equation may be transformed, if we take into consideration (10) and (13) and also the fact that at $H_m < H_L$ both the parts of the 3rd subgroup and the 3rd group are absent. We then obtain:

$$I'_0 = I_{0m} - \sum_i \left\{ (I_{0i})_m - \left[I'_0 + \frac{1}{N_i}(H + h_{0i}) \right] \right\} v_i'^{(2)}. \qquad (16')$$

The limits of the sum in (16′) depend on the values of H and I'_0, and on H_m and I_{0m}. This sum is extended over all the parts of the 2nd subgroup. The values of h_{0i} and N_i of these parts must be bound by inequality (15). Substituting into this expression the values of $(I_{0i})_m$ from (11), it is not difficult to transform it into the following relation:

$$h_{0i} \leqslant \frac{H_m - H}{2} + N_i \frac{I_{0m} - I'_0}{2}. \qquad (17)$$

The sum in (16′) must be extended over all the parts for which inequality (17) holds. Employing (16′), into which we substitute the value of $(I_{0i})_m$ from (11), we may obtain:

$$\frac{I_{0m} - I'_0}{2} = \sum_i \left[\frac{I_{0m} - I'_0}{2} + \frac{1}{N_i} \left(\frac{H_m - H}{2} - h_{0i} \right) \right] v_i'^{(2)}. \qquad (18)$$

A comparison of (18) and (10) with a consideration of (12) and (17), which determine the limits of the summation, will make evident the fact that the quantities $\frac{H_m - H}{2}$ are related to the quantities $\frac{I_{0m} - I'_0}{2}$ by expressions similar to those, by which the values of H are bound to the corresponding values of I_0. If there exists between I_0 and H a dependence of the type

$$I_0 = f(H) \quad \text{at} \quad H \geqslant (h_{0i})_{\min},$$

$$\text{then} \quad \frac{I_{0m} - I'_0}{2} = f\left(\frac{H_m - H}{2} \right) \text{ at } H_m - H \geqslant 2\,(h_{0i})_{\min}. \qquad (19)$$

From (19) it follows that

$$I'_0 = I_{0m} - 2f\left(\frac{H_m - H}{2} \right) *. \qquad (20)$$

Formulae (19) and (20) are true not only for values of I_0 and $I_{0m} - I'_0$, but also for I and $I_m - I'$. We easily become convinced that this is indeed the case by substituting I_0 from (4) into (10), (12), (17) and (18). Therefore, if the dependence

$$I = F(H) \quad \text{at} \quad H \geqslant (h_{0i})_{\min}$$

is found, it implies that

$$I' = I'_m - 2F\left(\frac{H_m - H}{2} \right) \text{ at } H_m - H \geqslant 2\,(h_{0i})_{\min} \qquad (21)$$

Employing (21) it is possible to plot the hysteresis curves with the aid of magnetization curves. Lord Rayleigh [5] was the first to show

* In the previous paper [1] devoted to those cases in which the mutual magnetic interaction between the regions of the ferromagnetic may be neglected, we obtained the formula $I'_0 = I_{0m} - 2f(H)$ which, evidently, essentially differs from (20).

that for a large number of polycrystalline ferromagnetics the following equation is valid in the region of small I:

$$I = \chi_0 H + bH^2 \tag{22}$$

where χ_0 and b are constants depending upon the material used. Assuming that in the region, for which (22) is valid, $H < H_L$, theoretical formulae for hysteresis curves in weak fields may be obtained on the basis of (21).

Having determined with the aid of (22) $F\left(\frac{H_m - H}{2}\right)$ and substituting this quantity into (4), we shall have obtained

$$I' = I_m - \chi_0 (H_m - H) - \frac{b}{2} (H_m - H)^2 . \tag{23}$$

Hence, taking (22) into account, we may obtain the formula of residual magnetism:

$$I_r = b \frac{H_m^2}{2} \tag{24}$$

and of the coercive force

$$H_c = \frac{\chi_0}{b} \left[\left(1 + \frac{bH_m}{\chi_0}\right) - \sqrt{1 + 2\frac{bH_m}{\chi_0}} \right] . \tag{25}$$

At $H_m \ll \frac{\chi_0}{b}$ in agreement with (25) and (24)

$$H_c \approx \frac{1}{4} \frac{b}{\chi_0} H_m^2 = \frac{1}{2} \frac{I_r}{\chi_0} . \tag{26}$$

The work of hysteresis $W_h = 4H_m I_m - 2\int_0^{2H_m} (I_m - I')\, d\,(H_m - H)$ in order to be in accord with (23), must equal

$$W_h = \frac{4}{3} bH_m^3 . \tag{27}$$

These expressions are identical with the well-known empirical formulae of Lord Rayleigh ([5]), obtained simultaneously with the main formula (22)*.

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* An attempt to explain Rayleigh's formulae has been previously undertaken by Preisach. However, in his scheme Preisach believed that the single parts of the ferromagnetic are always magnetized to saturation either longitudinally, or transversely with respect to the direction of the magnetic field (it would be more correct to say that he considered only such parts). The scheme suggested by the author of the present paper differs in principle from Preisach's. Rayleigh's formula for the hysteresis loop is here reached as the result of assuming the possibility of gradual magnetization of the parts in the course of overcoming the action of the demagnetizing field. It must be mentioned, however, that our scheme does not exclude Preisach's scheme, if the latter is employed for separate domains of spontaneous magnetization within the parts.

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PHYSICS

ON THE CASCADE THEORY OF COSMIC SHOWERS

By S. BELENKY

(Communicated by S. I. Wawilow, Member of the Academy, 22. XII. 1940)

The passage of radiation consisting of fast electrons* and hard photons through matter essentially involves the following processes: loss of energy by radiation («Bremsestrahlung»), creation of pairs by photons, energy losses by ionization, and the Compton-effect. A theory of this phenomenon, accounting for the energy loss by radiation and pair formation by photons, has been worked out in detail by Bhabha and Heitler (1), Carlson and Oppenheimer (2), Snyder (3), Landau and Rumer (4), Iwanenko and Sokolov (5), and many others. These authors have developed a mathematical method which can be most conveniently applied to the solution of this problem. However, the absorption of shower particles, whose rôle in the process is of prime importance and which is due in the first line to losses by ionization, has been estimated by the modern cascade theory either in a roughly approximate manner, or else with reference to special cases only. It is true that the cascade curve plotted by Snyder (3) and Serber (6) accounts also for losses by ionization, but the solution obtained by these authors is not convincing mathematically. Besides, they have failed to obtain the energy spectrum of electrons in a shower, having thus solved but a part of the problem. Arley's (7) spectrum adopted in the theory has been calculated on the assumption, that above a «critical» energy E_c, which is typical of a given material, there are taking place only processes of energy loss by radiation and formation of pairs, whereas below this energy processes of loss by ionization alone are going on. Arley has found the number of particles with energies below the critical one, taking into account the transition of electrons from the energy range $> E_c$ to the energy range $< E_c$, due to energy emission by these electrons. There

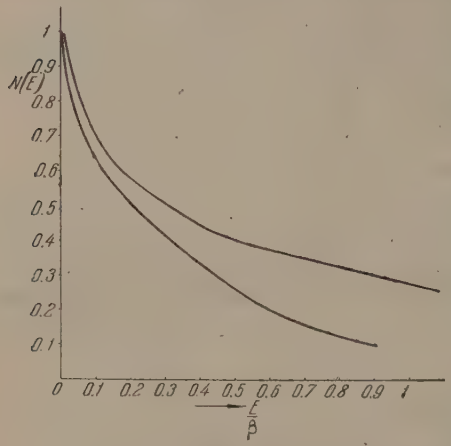

Fig. 1.

* Here, as well as in what follows, we understand by electrons both negative and positive electrons (positrons).

are three reasons why we consider Arley's spectrum to be incorrect. First, the number of electrons with energies $> E_c$ has been strongly overestimated because of the neglect of ionization losses. Secondly, Arley did not take into account those low energy electrons which are produced by photons of energies both above and below E_c. These electrons, however, considerably increase the number of particles with energies $< E_c$. The third reason is that Arley's calculation itself is lacking consistency. A consistent calculation made after Arley's method would inevitably lead to a spectrum showing a break at the point E_c. Thus, if we assume a sudden setting in of ionization losses, we obtain a spectrum of a discontinuous type. This means that Arley's method does not hold and that no correct results can possibly be obtained, unless the equations of the cascade theory will be solved taking due account of ionization losses, too. This problem has been solved by Tamm and the present author (8) for a spectrum of electrons in equilibrium averaged along the whole cascade curve, i. e. for a special case. The present article suggests a more general solution of the problem.

The fundamental equations of the cascade theory which account for energy losses by radiation, pair formation and ionization losses, have the following form:

$$\frac{dP(E,t)}{dt} = 2\int_E^{\infty} \Gamma(u)\, W_p(u,E)\, du + \int_E^{\infty} P(u)\, W_e(u, u-E)\, du - \int_0^E P(E)\, W_e(E,u)\, du + \beta \frac{dP}{dE} \tag{1}$$

$$\frac{d\Gamma(E,t)}{dt} = \int_E^{\infty} P(u)\, W_e(u,E)\, du - \int_0^E \Gamma(E)\, W_p(E,u)\, du. \tag{2}$$

The equations are written in length units adopted in the cascade theory [cf. (8) or (6)]. $P(E,t)\,dE$ and $\Gamma(E,t)\,dE$, respectively, are the numbers of electrons and photons at a given depth t having energies between E and $E+dE$. $W_e(E,E')\,dE'\,dt$ and $W_p(E,E')\,dE'dt$, respectively, are the probabilities that an electron of energy E, after having passed through a layer of matter dt, will emit a photon of an energy E', and that a photon of energy E will produce a pair of electrons having energies between E' and $E-E'$. In the sequel we shall make use for these probabilities of the simplified expression introduced by Carlson and Oppenheimer (2).

$$W_e(E,E') = \frac{1}{E'}\,; \quad W_p(E,E') = \frac{\sigma}{E}\,; \quad \sigma \cong \frac{2}{3}\,. \tag{3}$$

However, in the treatment of our final results we shall take into account also the more exact expressions calculated for these magnitudes by Bartlett (6).

Ionization losses are described by the last term of equation (1):

$$\beta = \frac{\pi mc^2}{2\alpha Z}\,\frac{\ln \vartheta}{\ln\left(191 \cdot Z^{-1/3}\right)}\,; \quad \vartheta = \frac{2}{\alpha^4 Z^2}\left(\frac{E}{mc^2}\right)^3. \tag{4}$$

β is considered as a magnitude independent of energy, and is estimated for the main energy of electrons in a shower.

Let us assume that a shower is produced by an electron of energy E_0, incident upon a layer of matter. The boundary conditions of our problem will thus be as follows:

$$\Gamma(E, t=0) = 0; \quad P(E, t=0) = \delta(E - E_0). \tag{5}$$

Let us now apply to the functions $P(E,t)$ and $\Gamma(E,t)$ Laplace-Mellin's transformation:

$$P(E,t)=\frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} ds\,P(s,t)\,E^{-s-1};\quad P(s,t)=\int_0^{\infty} E^s\,P(E,t)\,dE \tag{6}$$

and the transformation of Fourier:

$$P(s,t)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dk\,P(s,k)\,e^{kt};\quad P(s,k)=\int_0^{\infty} P(s,t)\,e^{-kt}\,dt. \tag{7}$$

Now, let us multiply (1) and (2) first by E^s and integrate from 0 to ∞ over E, after which let us multiply them by e^{-kt} and integrate from 0 to ∞ over t, boundary conditions being taken into account. We obtain the following equations for $P(s,k)$ and $\Gamma(s,k)$:

$$k\Gamma(s,k)=-\sigma\Gamma(s,k)+\frac{1}{s}P(s,k) \tag{8}$$

$$-E_0^s+kP(s,k)=2\sigma\frac{\Gamma(s,k)}{s+1}-\{\psi(s+1)+\gamma\}\,P(s,k)-\beta sP(s-1,k). \tag{9}$$

Here $\psi(s)=\frac{d}{ds}\log s!$ while γ is Euler's constant. Eliminating $\Gamma(s,k)$ from (8) and (9), we obtain:

$$E_0^s=P(s,k)\left[k-\frac{2\sigma}{(s+1)\,s\,(k+\sigma)}+\{\psi(s+1)+\gamma\}\right]+\beta sP(s-1,k). \tag{10}$$

A formal solution of this equation can be obtained, but subsequent transformations involve considerable difficulties. We simplify this equation by replacing the two last terms in brackets by the following:

$$\frac{-q\sigma}{(s+1)\,s\,(k+\sigma)}+\frac{qs}{(s+1)};\qquad q=2.4. \tag{11}$$

Such a simplification has already been suggested by Tamm and the present author [8]. It is similar to, though not identical with that introduced by Carlson-Oppenheimer. It refers to those terms of the equation which correspond to the energy losses by radiation and to the formation of pairs. Within the range of very high energies, where only these processes are essential, with the aid of this simplified equation we obtain a solution which differs but little from the rigorous solution [8]. Losses by ionization being accurately estimated, our solution is likely to be valid for any energy. It should be noted, moreover, that the cascade curve obtained by our method scarcely differs from that by Serber [6], who made use of exact equations. Selecting the numerical coefficient for q, we took the expressions for W_p and W_e suggested by Bartlett [7]. The solution of equation (10) has the following form:

$$P(s,k)=\left(\frac{E_0}{\beta}\right)\sum_0^{\infty} n\,E_0^s\left[-\frac{E_0}{\beta}(k+q)\right]^n\times$$

$$\times\frac{[\Gamma(s+1)]^2\,\Gamma(s+2)\,\Gamma(s+n-s_1)\,\Gamma(s+n-s_2)}{\Gamma(s-n+1)\,[\Gamma(s+n+2)]^2\,\Gamma(s-s_1)\,\Gamma(s-s_2)} \tag{12}$$

$$s_{1,2}=-\frac{2q+3k}{2\,(k+q)}\pm\sqrt{\left[\frac{2q+3k}{2\,(k+q)}\right]^2-\frac{2k}{(k+q)}\left(1+\frac{q}{2}\frac{1}{(k+\sigma)}\right)}. \tag{12'}$$

By means of transformations in a complex plane, $P(s,k)$ may be represented in the form of a series arranged in an increasing order of powers of $\left(\frac{\beta}{E_0}\right)$. Assuming the parameter $\left(\frac{\beta}{E_0}\right)$ small, we make use of the

first term of this series $P_0(s,k)$, which we do not write. The analysis of the next terms of the expansion presents no difficulties, and they prove unessential even with $\frac{\beta}{E_0} \cong 0.1$. $P_0(s,k)$ and $P_0(E,t)$ are connected by the following relation:

$$P_0(E,t) = -\frac{1}{4\pi^2} \int_{d-i\sim}^{d+i\sim} ds \int_{-i\sim}^{+i\sim} dk P_0(s,k) E^{-s-1} e^{kt}. \qquad (13)$$

It appears possible to integrate this expression over the variable s. After having integrated over E, we obtain the following expression for $N(E,t)$—the number of electrons of energies above E, at a given depth t:

$$\left.\begin{aligned} N(E,t) &= \frac{1}{2\pi i} \int_{-i\sim}^{+i\sim} dk n(E,k) e^{kt} \\ n(E,k) &= \varepsilon_0^{s_1+1} f(k) \int_0^{\sim} dx\, x e^{-x} \int_0^1 dy \left(y + \frac{\varepsilon}{x}\right)^{-(s_1+1)} y^{s_1+1} (1-y)^{-(s_2+2)} \\ \varepsilon &= \frac{E}{\beta}(k+q);\ \varepsilon_0 = \frac{E_0}{\beta}(k+q);\ f(k) = \frac{\Gamma(s_1-s_2)}{(k+q)[\Gamma(s_1+2)]^2 \Gamma(-s_2-1)} \end{aligned}\right\} \qquad (14)$$

Let us examine some special cases.

1) With $\varepsilon >> 1$

$$N(E,t) = \frac{1}{2\pi i} \int_{-i\sim}^{+i\sim} dk \frac{s_1+2}{(k+q)(s_1-s_2)} \left(\frac{E_0}{E}\right)^{s_1+1} e^{kt}. \qquad (15)$$

It can easily be shown that the expression obtained coincides with the solution of the fundamental equation of the theory for the case when losses by ionization are neglected.

2) Making use of Fourier's transformation we find from (14):

$$n(E,0) = \int_0^{\sim} N(E,t)\, dt.$$

Computing $n(E,0)$ with the help of (12′), we have:

$$n(E,0) = \frac{E_0}{\beta}\{1 + \varepsilon e^{\varepsilon} \mathcal{E}_i(-\varepsilon)\}; \quad \varepsilon = \frac{E}{\beta} q;\ \mathcal{E}_i(-\varepsilon) = -\int_{\varepsilon}^{\sim} \frac{e^{-x}}{x} dx. \qquad (16)$$

Expression (16) agrees with an accuracy to the term of the order $e^{-\frac{E_0}{\beta}}$ with the formula for a spectrum integrated over depth, that has been calculated by Tamm and the author (8).

3) Putting ε in (14) equal to 0, we obtain the full number of electrons $N(0,t)$ in a shower, as a function of depth:

$$N(0,t) = \frac{1}{2\pi i} \int_{-i\sim}^{+i\sim} \frac{\Gamma(s_1-s_2)}{[\Gamma(s_1+2)]^2 \Gamma(-s_2)} \frac{1}{(k+q)} \varepsilon_0^{s_1+1} e^{kt} a k. \qquad (17)$$

Estimating this expression by the saddle point method, we obtain a cascade curve deviating but little, as has been stated above, from that of Serber *.

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* To find out the relation between the method of Serber and Snyder and our own method, we have analyzed by both methods the so-called «Furry model» which does not account for the difference between electrons and photons, while it is being assumed that for each

From expression (14) it is possible to obtain the spectrum of electrons at any depth. Integration over the variable k is easily performed by the saddle point method; while integration over x and y for the general case has to be made numerically. We give here the spectra of electrons near the maximum and at a depth $t \cong 15$. The upper curve in Fig. 1 represents the electron spectrum in the neighbourhood of the peak of the cascade curve, the lower one at $t \cong 15°$. For a unit we take the number of electrons with energies above zero. The latter spectrum has been calculated by O. N. Wawilow. The spectrum of photons in a shower can be obtained in a similar way.

Substituting $P_0(s, k)$ [or $\Gamma_0(s, k)$] in the first expression of (7), it becomes possible to calculate $P_0(s, t)$ [or $\Gamma_0(s, t)$]. Putting in this expression s equal to 1, we find, as may be seen from the second expression of (6), the full energy of electrons or photons in a shower. With t and $\ln \frac{E_0}{\beta}$ sufficiently large, we obtain relatively simple expressions for the magnitude $\frac{E_{el}(t)}{N(t)}$ and $\frac{E_{ph}(t)}{N(t)}$, where $E_{el}(t)$ and $E_{ph}(t)$ are the full energies of electrons and photons, respectively, while $N(t)$ is the full number of electrons at a given depth t. Thus, e. g., with $t \cong 2.8 \ln \frac{E_0}{\beta}$ $\frac{E_{el}(t)}{N(t)} = 0.6\beta$, $\frac{E_{ph}(t)}{N(t)} = 1.6\beta$, while with $t \cong 4.4 \ln \frac{E_0}{\beta}$, $\frac{E_{el}(t)}{N(t)} = 0.38\beta$ and $\frac{E_{ph}(t)}{N(t)} = 1.5\beta$.

In conclusion I wish to express my gratitude to Prof. Ig. Tamm for many a valuable advice in the course of this work.

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particle there exists a definite probability of fission. Contrary to Nordsiek, Lamb and Uhlenbeck[9], ionization losses have been taken into account by us. If the first term in the expansion over $\frac{3}{E_0}$ be taken again, our solution proves to coincide with that obtained by the method of Snyder. However, in the case of Furry's model we also failed to obtain by Snyder's method an obvious energy spectrum of the particles.

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TECHNISCHE PHYSIK

UNTERSUCHUNGEN AM ELEKTRONENSTRAHLGENERATOR MIT ELEKTRISCHEM BREMSFELD

Von S. GVOSDOVER, L. LOSCHAKOW und J. TERLEZKI

(Vorgelegt von S. I. Wawilow, Mitglied der Akademie, den 15. XII. 1940)

In der Arbeit werden Versuche mit Elektronenstrahlgeneratoren dargelegt, deren Prinzip von Hahn ([1], [2]) entwickelt worden ist.

Die bei unseren Versuchen benutzte Elektronenstrahlröhre besteht aus einem Metallzylinder, der in einer Glasröhre quer zu deren Achse angeordnet ist. Im Zylinder sind in der Nähe seines verschlossenen Bodens zwei sich diametral gegenüberliegende Schlitze zum Einlassen des Elektronenstrahls vorhanden. Innerhalb des Zylinders durchlaufen die Elektronen ein kurzes Rohr, das von einem längs der Zylinderachse angebrachten Drahte getragen wird. Das ganze Elektrodensystem ist als ein koachsialer konzentrischer Kabel ausgebildet, der durch die Glaswandungen der Röhre nach aussen geleitet wird. Ausserhalb der Röhre wird der Zuleitungskabel durch die bewegliche Scheibe eingeschaltet. Der von der Kathode ausgehende Elektronenstrahl tritt durch die Schlitze des Zylinders aus und gelangt nach dem Verlassen des Zylinders in das elektrische Bremsfeld, das von einer besonderen Reflektorelektrode erzeugt wird. Ein Teil der Elektronen wird von dem Reflektor durch den Zylinderschlitz zur Kathode zurückgeworfen. Alle Versuche wurden mit Vakuumröhren vorgenommen, da diese stabiler als die gasgefüllten Röhren funktionieren, obschon wir mit den letzteren bedeutend intensivere Schwingungen erregen konnten.

Die Entstehung von Schwingungen machte sich bemerkbar durch heftige Stromänderung im Reflektorkreis bei Abstimmung des Schwingungssystems, die durch Scheibeverschiebung besorgt wird. Zur Messung der Wellenlänge diente ein Lecher-System, das mit einer Drahtschleife des konzentrischen Kabels induktiv gekoppelt ist. Die Intensität der Schwingungen konnte mit Hilfe eines dem Lecher-System angekoppelten Indikators beurteilt werden.

Der Prüfung unterlagen mehrere Röhren mit verschiedenen Abmessungen ihrer Innenteile und verschiedener Art der Ableitung des Schwingungssystems aus der Röhre in den Aussenraum. Es erwies sich, dass die kapazitive Kopplung zwischen dem Zylinder in der Röhre und der Aussenhülle der äusseren konzentrischen Linie bei kompaktem Innendraht in einigen Fällen zu intensiveren Schwingungen führt, als die galvanische Kopplung.

Die Messungen ergaben, dass die Erregung nur bei einigen Abstimmungsintervallen des Schwingungssystems und bei bestimmten Grössenbereichen der Zylinder- und Reflektorspannungen zustande kommt. Innerhalb der

Erregungsgebiete folgt die Wellenlänge der Schwingungen den jeweiligen Änderungen in der Abstimmung des Schwingungssystems.

Nachstehend werden die an einer von den geprüften Röhren gewonnenen Versuchsergebnisse wiedergegeben. In der Tabelle sind die Messergebnisse zusammengefasst, die sich auf die Abhängigkeit der erregten Wellenlänge von der Abstimmung des Schwingungssystems bei konstanter Zylinderspannung beziehen.

Spannung am Zylinder und Innendraht (in Volt)		Grenzwerte der Länge der äusseren konzentrischen Linie l und der Wellenlänge λ (in cm)		
820	l	9.5—11.6	—	—
	λ	62 —72	—	—
880	l	9.5—10.5	—	17.5—19.5
	λ	62 —66	—	29 —31
1060	l	—	14.8—16.0	—
	λ	—	26 —27	—

In der Tabelle sind die Grenzwerte der Länge der äusseren konzentrischen Linie l und der Wellenlänge der Schwingungen λ in verschiedenen Erregungsgebieten angegeben. Bei anderen Abstimmungen traten keine Schwingungen auf. Jeder Zylinderspannung wurde eine Reflektorspannung derart zugeordnet, damit die Schwingungsintensität ihren Maximalwert erreichte.

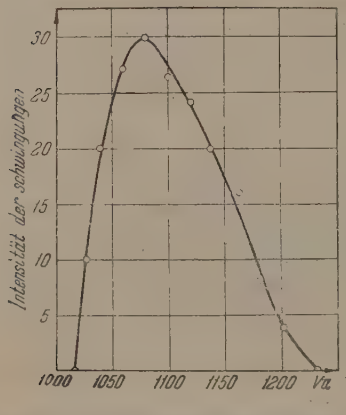

Abb. 1. Abhängigkeit der Schwingungsintensität von der Zylinderspannung bei dem äusseren Schwingungssystem von 16 cm Länge. Erregte Wellenlänge $\lambda = 26$ cm.

Die Tabelle lässt auf das Vorhandensein bestimmter Erregungsgebiete schliessen, innerhalb derer die Wellenlänge von der Abstimmung des Schwingungssystems abhängt. Wie aus der Tabelle ersichtlich, existieren bei der Zylinderspannung von 880 Volt zwei Erregungsgebiete, in denen Schwingungen auftreten: das erste Gebiet erstreckt sich von $\lambda = 62$ cm bis $\lambda = 66$ cm, das zweite von $\lambda = 29$ cm bis $\lambda = 31$ cm. Bei geringerer Zylinderspannung (820 Volt) existiert nur ein Erregungsgebiet von $\lambda = 62$ cm bis $\lambda = 72$ cm. Bei höherer Zylinderspannung (1060 Volt) lässt sich ein Erregungsgebiet von $\lambda = 26$ cm bis $\lambda = 27$ cm beobachten. Mit steigender Zylinderspannung verschieben sich die Erregungsgebiete den kürzeren Wellenlängen zu.

Die bei 1060 Volt auftretenden Wellenlängen, sowie die kürzeren bei 880 Volt erregten Wellen von ca 30 cm, entsprechen annähernd den Werten, die nach den von Hahn und Metcalf [3] entwickelten Formeln berechnet werden können. Dagegen lassen sich die 880 Volt und 820 Volt auftretenden längeren Wellen von ca 60 cm nicht durch solche theoretische Erwägungen erklären. Die Theorie von Hahn und Metcalf[3] lässt Erregungsgebiete zu, deren Wellenlängen sich im Verhältnis mit der ungeraden Zahlenreihe ändern (1 : 3 : 5 ...). Unsere mit mehreren Röhren vorgenommenen Versuche haben dagegen gezeigt, dass bei Vorhandensein zweier Erregungsgebiete die zugehörigen Wellenlängen sich annähernd wie 1 : 2 verhalten.

Die weitere Untersuchung galt der Abhängigkeit der Schwingungsintensität von der Zylinderspannung bei konstanter Abstimmung des Schwingungssystems und konstanter optimaler Reflektorspannung. Aus den in Abb. 1 wiedergegebenen Messergebnissen geht hervor, dass bei einer genau fixierten Abstimmung des Schwingungssystems die Schwingungen nur in einem bestimmten Grössenbereich der Zylinderspannungen auftreten. Wie ferner erwiesen, übt die Änderung der Zylinderspannung im Erregungsbereich keinen bemerkbaren Einfluss auf die erregte Wellenlänge aus.

Als Gegenstand besonderer Untersuchung war auch die Abhängigkeit des Gleichstroms im Reflektorkreis von der Abstimmung des Schwingungssystems bei konstanten Elektrodenspannungen. Bei geringen Reflektorspannungen von der Grössenordnung + 4 Volt liess sich ein klar ausgeprägter Detektoreffekt wahrnehmen, der sich im steilen Anstieg des Gleichstromes im Reflektorkreis beim Auftreten der Schwingungen äusserte.

Die Werte der Reflektorspannungen, bei denen die Intensität der Schwingungen ihr Maximum erreicht, übertreffen diejenigen Werte, bei welchen die grösste Detektorwirkung auftritt. Die kürzeste Wellenlänge, die wir mit Hilfe einer unserer Röhren erhalten konnten, betrug bei einer Zylinderspannung von 1300 Volt 16 cm. Von Interesse ist, dass das andere Erregungsgebiet dieser Röhre bei derselben Spannung in der Nähe von $\lambda = 36$ cm liegt.

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GEOPHYSIK

ÜBER DIE SCHALLSTREUUNG IN DER TURBULENTEN STRÖMUNG

Von A. M. OBUKHOV

(Vorgelegt von O. J. Schmidt, Mitglied der Akademie, den 7. XII. 1940)

Die Erforschung der Gesetze der Schallstreuung unter den Verhältnissen der turbulenten Atmosphäre ist ein aktuelles Problem der meteorologischen Akustik. In der vorliegenden Mitteilung ist auf Grund der Gleichungen der Akustik des bewegten Mittels der Versuch einer angenäherten Lösung des Problems mit Hilfe der Methoden der statistischen Theorie der Turbulenz gemacht worden.

§ 1. Die Grundgleichung der Schallausbreitung im bewegten Mittel kann in folgender Form geschrieben werden:

$$\Delta\varphi - \frac{1}{c^2} D_t^2 \varphi = 0 \tag{1}$$

wo φ das Potential der Schallwelle* ist und

$$D_t = \frac{\partial}{\partial t} + (\bar{u}\,\overline{V});$$

D_t ist der Operator der vollständigen Differentiation nach der Zeit; $\bar{u}$, der Vektor der Bewegungsgeschwindigkeit der Flüssigkeit; c, die Schallgeschwindigkeit. Die Aufstellung dieser Gleichung unter Annahme einiger vereinfachender Voraussetzungen ist von Andrejev(1) durchgeführt worden.

Nehmen wir an, dass die mittlere Strömungsgeschwindigkeit gleich 0 ist und $\bar{u}$ den augenblicklichen Wert der Pulsationsgeschwindigkeit in der turbulenten Strömung darstellt. Da die Pulsationsgeschwindigkeiten unter den Bedingungen der Erdatmosphäre im Vergleich mit der Schallgeschwindigkeit klein sind, so werden wir in den Gleichungen nur diejenigen Glieder stehen lassen, die von der kleinsten Potenz der kleinen Grösse $\mu = \frac{u}{c}$ sind, indem wir die Glieder höherer Ordnungszahl vernachlässigen.

Indem wir den Ausdruck für den Operator D_t^2 ausschreiben und in der Gleichung (1) die Glieder, die von der zweiten Ordnung klein sind, vernachlässigen, erhalten wir die «verkürzte» Gleichung der Schallausbreitung:

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{2}{c^2}\left(\bar{u},\ \operatorname{grad}\frac{\partial\varphi}{\partial t}\right) \tag{2}$$

von der wir im folgenden Gebrauch machen werden.

* In der vorliegenden Arbeit vernachlässigen wir die potentiallose Komponente des Schallfeldes, die, im allgemeinen, bei der Ausbreitung des Schalles in einer wirbelhaften Strömung vorhanden ist. Die Bedeutung dieser Komponente im Vergleich zur potentialen Komponente bei den der Atmosphäre entsprechenden Bedingungen, ist aber nicht gross.

§ 2. In bezug auf das Feld der turbulenten Pulsationen werden wir folgendes voraussetzen:

1) Die Verteilung der Pulsationen im Raume ist von zufälligem Charakter. In genügend weit voneinander entfernten Punkten sind die Pulsationen statistisch unabhängig.

2) Die Verteilung der Pulsationen im Raume ist statistisch stationär; die Beziehungsmomente zwischen den Pulsationen in verschiedenen Punkten hängen nur von dem Verbindungsvektor dieser Punkte ab:

$$E\bar{u}(\bar{r}_1)\bar{u}(\bar{r}_2) = M(\bar{r}_1 - \bar{r}_2) \tag{3}$$

wo E das Symbol der mathematischen Erwartung und M der Tensor der Beziehungsmomente ist.

3) Das Feld der turbulenten Pulsationen ist im Sinne von Karmann [2,3] isotrop.

Die Aufgabe der vorliegenden Mitteilung besteht in der Aufstellung der «lokalen» Gleichung der Schallstreuung, die die Transformation der Schallwelle durch irgendein «Elementarvolumen» der turbulenten Strömung bestimmt. Greifen wir aus dem Innern der turbulenten Strömung ein Element von der Form eines Würfels des Volumens V, dessen Abmessungen folgenden Bedingungen entsprechen müssen, heraus: A) Im Vergleich zu den Abmessungen der turbulenten Störungen muss das Volumen genügend gross sein, so dass die Korrelation in der Entfernung ungefähr eines Viertels der Seitenlänge des Würfels praktisch verschwindet. B) Die Abmessungen des Elementarvolumens sind von solcher Grösse, dass die Zahl der Schallwellen, die in ihm Platz finden, kleiner als $\frac{1}{4\mu}$ ist, wo $\mu = \frac{u}{c}$ das Verhältnis des mittleren Quadrates der Pulsationsgeschwindigkeit zur Schallgeschwindigkeit ist.

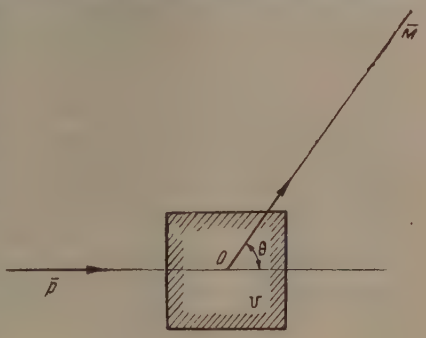

Abb. 1.

Unter gewöhnlichen Verhältnissen in der Atmosphäre ist $\mu \approx 0.003$ bis 0.001, $\frac{1}{\mu} \approx 300$ bis 1000.

Der vorgenommenen *lokalen Aufgabe* entsprechend, werden wir unter Vorbehalt annehmen, dass die Pulsationsgeschwindigkeit ausserhalb des betrachteten Raumes $\bar{u} \equiv 0$ ist.

§ 3. Die durchgehende Welle sei durch die Gleichung gegeben:

$$\varphi_0(\bar{r}, t) = A_0 e^{j(\omega t - t(\bar{p}, \bar{r}))} \tag{4}$$

wo $j = \sqrt{-1}$, $\frac{\omega}{p} = c$, $\bar{p}$ — der Wellenvektor and A_0 — die Amplitude sind.

Wir werden die Lösung der Gleichung (2), d. h. die Funktion $\varphi(\bar{r}, t)$ suchen, die sich bei Entfernung vom «streuenden» Elemente V asymptotisch der Funktion $\varphi_0(\bar{r}, t)$ nähert, die die ungestörte Welle definiert. Angenommen es sei $\varphi = \varphi_0 + \psi$. Dann erhalten wir für die «Störung» ψ die Differentialgleichung

$$\Delta\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t} = \frac{2}{c^2}(\bar{u}, \operatorname{grad}\varphi_0) + \frac{2}{c^2}(\bar{u}, \operatorname{grad}\psi). \tag{5}$$

Die Störung verschwindet im Unendlichen: $\psi(\bar{r}, t) \to 0$ für $r \to \infty$.

Indem wir die inhomogene Wellengleichung (5) mittels der Methode der retardierten Potentiale integrieren, gelangen wir zu folgender Integralgleichung:

$$\psi(\bar{r}, t) = -\frac{1}{2\pi c^2}\int\int\int_V \left(\frac{\bar{u}(\bar{\rho})}{R}, \operatorname{grad} \varphi_0\left(\bar{\rho}, t - \frac{R}{c}\right)\right) dV_{\bar{\rho}} -$$

$$- \frac{1}{2\pi c^2}\int\int\int_V \left(\frac{\bar{u}(\bar{\rho})}{R}, \operatorname{grad} \psi\left(\bar{\rho}, t - \frac{R}{c}\right)\right) dV_{\bar{\rho}} \qquad (6)$$

wo $R = |\bar{r} - \bar{\rho}|$ ist.

Legen wir das Zentrum O des Volumens V in den Koordinatenanfang; bezeichnen wir mit $\bar{m}$ den Einzelvektor, der die Richtung des zerstreuten Strahles bestimmt, und mit R_0 — die Entfernung des Beobachtungspunktes vom Zentrum O. Bei grossen Werten von R_0 gilt folgende Zerlegung:

$$R = R_0 - (\bar{m}, \bar{\rho}); \quad \frac{1}{R} = \frac{1}{R_0} + \frac{1}{R_0^2}(\bar{m}, \bar{\rho}).$$

Indem wir die erhaltene Integralgleichung nach der Methode der schriftweisen Annäherung lösen und den Parameter der kleinen Grösse μ wieder einführen, erhalten wir die gesuchte Lösung in Form einer Reihe:

$$\psi = \psi_1 + \psi_2 + \psi_3 + \ldots; \quad \psi_2 = O(\mu^2); \quad \psi_3 = O(\mu^3) \ldots$$

Die Konvergenz des Vorganges ist bei Einhaltung der Bedingung «B» sichergestellt. Beschränken wir uns in Anbetracht der Kleinheit der Grösse μ auf die erste Annäherung, so erhalten wir dafür bei grossen Werten von R_0 den folgenden Ausdruck:

$$\psi_1(R_0, \bar{m}, t) = \frac{A_0\omega}{2\pi c^2 R_0}\left(\bar{p}, \int\int\int_V \bar{u}(\bar{\rho}) e^{j\omega(t-t_0) - j(\bar{p}-p\bar{m}, \bar{\rho})} dV_{\bar{\rho}}\right) =$$

$$= \frac{A_0\omega}{2\pi c^2 R_0} e^{j\omega(t-t_0)}\left(\bar{p}, \int\int\int_V \bar{u}(\bar{\rho}) e^{-j(\bar{p}-p\bar{m}, \bar{\rho})} dV_{\bar{\rho}}\right) \qquad (7)$$

wo $t_0 = \frac{R_0}{c}$ ist.

Für grosse Entfernungen vom Zentrum O ist die Wirkung der Zerstreuung der Schallwelle durch das Element V der Ausstrahlung der sphärischen Welle äquivalent. Da die Pulsationen $\bar{u}(\bar{\rho})$ von zufälligem Charakter sind, ist die durch die Funktion ψ_1 dargestellte Welle mit der durchgehenden Welle φ_0 nicht kohärent.

§ 4. Für die Berechnung der Schallenergie, die durch das Element V in der Richtung des Vektors $\bar{m}$ zerstreut worden ist, bestimmen wir das quadratische Mittel der Amplitude ψ_1:

$$E|\psi_1|^2 = E\psi_1\psi_1^* =$$

$$E\frac{A_0^2\omega^2}{4\pi^2c^4R_0^2}\left(\bar{p}, \int\int\int_V\int\int\int_V \bar{u}(\bar{\rho}_1)\bar{u}(\bar{\rho}_2) e^{-j((\bar{p}-p\bar{m})(\bar{\rho}_1-\bar{\rho}_2))} dV_{\bar{\rho}_1} dV_{\bar{\rho}_2}\bar{p}\right) =$$

$$= \frac{A_0\omega^2}{4\pi^2c^4R_0^2}\left(\bar{p}, \int\int\int_V\int\int\int_V M(\bar{\rho}_1 - \bar{\rho}_2) e^{-j((\bar{p}-p\bar{m})(\bar{\rho}_1-\bar{\rho}_2))} dV_{\bar{\rho}_1} dV_{\bar{\rho}_2}\bar{p}\right) \qquad (8)$$

wo $M(\bar{\rho}_1 - \bar{\rho}_2) = E\bar{u}(\bar{\rho}_1) \cdot \bar{u}(\bar{\rho}_2)$ der Tensor der Beziehungsmomente ist.

Unter Benutzung der Bedingung «A» kann man ohne grossen Fehler die erste Integration über das Volumen V durch die Integration über den ganzen Raum ersetzen. Führen wir die zusätzliche Bezeichnung

$$\Phi(\bar{k}) = \frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty}\int\int e^{-j(\bar{k}\bar{r})} M(\bar{r}) dV_{\bar{r}} \qquad (9)$$

ein, so erhalten wir den endgültigen Ausdruck für die mathematische Erwartung $|\psi_1|^2$:

$$E|\psi_1|^2 = 2\pi \frac{A_0^2}{c^4} \cdot \frac{\omega^2}{R_0^2} (\bar{p}, \Phi(\bar{p} - \bar{pm})\bar{p}) V_0 \qquad (10)$$

wo V_0 das Volumen des zerstreuenden Elementes V ist. Die Tensorfunktion $\Phi(k)$ kann man die «Spektralfunktion» der turbulenten Strömung nennen. Sie ist eng mit der Verteilung der turbulenten Pulsationen unter den Frequenzen verbunden. Für den Fall des inkompressiblen isotropen Pulsationsfeldes wird die Spektralfunktion nur durch eine Skalarfunktion der folgenden Tensorgleichung definiert:

$$\Phi(\bar{k}) = E\bar{u}^2 \left(F(k) I - F(k) \frac{k \cdot \bar{k}}{k^2} \right) \qquad (11)$$

wo I das Symbol des einzelnen Tensors, $E\bar{u}^2$ — das mittlere Quadrat der Pulsationsgeschwindigkeit sind.

Führen wir den Streuungswinkel θ, den Winkel zwischen der Richtung der durchgehenden Welle und der Richtung des zerstreuten Strahles $\bar{m}$, ein. Dann erhalten wir nach einfachen Umformungen aus der Gleichung (10) mit Hilfe der Gleichung (11)

$$E|\psi_1|^2 = \frac{A_0^2}{R_0^2} \cdot \frac{2\pi\omega^2}{c^4} \cdot \frac{\bar{u}^2}{c^2} \cdot F\left(4 \frac{\omega}{c} \sin^2 \frac{\theta}{2}\right) \cos^2 \frac{\theta}{2}. \qquad (12)$$

Auch ist es bequem mit Benutzung des Begriffs des linearen Maßstabes der Turbulenz eine dimensionslose Spektralfunktion f einzuführen. Zu diesem Zweck setzen wir

$$F(k) = l^3 f(lk)$$

wo l irgendeine charakteristische Länge ist, die mit der Natur der turbulenten Strömung verknüpft ist. Gleichung (12) kann folgendermassen umgeschrieben werden:

$$E|\psi_1|^2 = \frac{A_0^2}{R_0^2} (2\pi)^5 \left(\frac{l}{\lambda}\right)^4 \mu^2 f\left(8\pi \frac{l}{\lambda} \sin^2 \frac{\theta}{2}\right) \cos^2 \frac{\theta}{2} \cdot \frac{V}{l} \qquad (13)$$

$$\mu^2 = \frac{E(\bar{u}^2)}{c^2}.$$

Die Schallenergie ist der zweiten Potenz der Amplitude proportional. Deswegen wird in dem Element des Raumwinkels $d\Omega$ eine Energiemenge zerstreut, die $E|\psi_1|^2 R_0^2 d\Omega$ proportional ist.

Setzen wir $V_0 = S\Delta x$, wo Δx ein Element des von der einfallenden Welle durchlaufenen Weges ist. Bezeichnen wir mit U die Energieströmung, so ist $U_0 = \varkappa A_0^2 S$ die durchgehende Energieströmung, $U' = \varkappa \int\int_{\Omega} |\psi^2 \; R_0^2 d\Omega$ — die Strömung der zerstreuten Energie, $\varkappa$ — der Koeffizient der Proportionalität. Bei der Multiplikation der beiden Teile der Gleichung (13) mit $\varkappa R_0^2 d\Omega_\theta$ erhalten wir die in der Zeiteinheit in dem Element des Raumwinkels $d\Omega$ in der Richtung von θ zerstreute Menge der Schallenergie

$$dU' = U_0 (2\pi)^5 \mu^2 \left(\frac{l}{\lambda}\right)^4 f\left(8\pi \frac{l}{\lambda} \sin^2 \frac{\theta}{2}\right) \cos^2 \frac{\theta}{2} \frac{\Delta x}{l} d\Omega_\theta. \qquad (14)$$

Indem wir über die Sphäre integrieren, erhalten wir die gesamte Menge der zerstreuten Energie

$$U' = U_0 \mu^2 f_1 \left(\frac{l}{\lambda}\right)^4 \frac{\Delta x}{l}, \qquad (15)$$

wo $f_1\left(\frac{l}{\lambda}\right) = (2\pi)^5 \int\int_{\Omega} f\left(8\pi \frac{l}{\lambda} \sin^2 \frac{\theta}{2}\right) \cos^2 \frac{\theta}{2} d\Omega_\theta$

ist. Die Gleichungen (14) und (15) geben eine vollständige Lösung der Aufgabe.

Es ist naheliegend die Hypothese aufzustellen, dass die zerstreute Energie U' der Grundströmung der Energie U_0 entnommen wird, weshalb die Grundströmung U_0 bei der Durchdringung der Dicke der turbulenten Schicht der Atmosphäre eine Dämpfung erfährt.

Der Dämpfungskoeffizient der Energie der Grundströmung auf Kosten der Streuung lässt sich folgendermassen ausdrücken:

$$q = \mu^2 \left(\frac{l}{\lambda}\right)^4 f_1\left(\frac{l}{\lambda}\right)\frac{1}{l}.$$

Um zahlenmässige Ergebnisse zu erhalten, ist die Kenntnis der dimensionslosen spektralen Funktion f, des Wertes des Turbulenzmasstabes l und des mittleren Quadrates der Pulsation nötig. Diese Angaben können mit Hilfe von geeignet angestellten experimentellen Untersuchungen unter natürlichen atmosphärischen Verhältnissen gewonnen werden.

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CHEMISTRY

A GRAPHICAL METHOD OF CALCULATING POLYELECTRODE ELECTROCHEMICAL SYSTEMS AS APPLIED TO CORROSION PROCESSES

By N. D. TOMASHOV

(Communicated by A. N. Frumkin, Member of the Academy, 6. XII. 1940)

To solve the problem of a polyelectrode element in the first approximation means making it possible to determine theoretically: which electrodes of a given polyelectrode system will act as anodes and which as cathodes. In order to solve this question more completely, it is necessary to include a quantitative current calculation for each electrode in the system.

But the problem has not only a purely theoretical interest. Its practical importance lies in the fact that the anodes will dissolve (corrode) at a rate proportional to the current acting on them, while the cathodes will remain in general unchanged. Once it is solved, we shall be able to draw important conclusions concerning the proper production of practical polyelectrode corrosion systems, as e. g. the structure and surface state of metallic alloys, or polymetallic constructions subject to corrosion in a solution of electrolytes.

The first attempt in this direction was made by Akimov ([1]) in 1930. The opinion expressed by him was briefly as follows: if one of the electrodes of the polyelectrode system has a potential more negative than half the sum of the potentials of the two electrodes with the most widely differing effective potentials in the given system, that electrode will act as anode, otherwise (i. e. if its potential is more positive than half the sum) it acts as cathode.

Further elaboration of the polyelectrode element problem, undertaken by Akimov together with the author of this paper ([2]), showed that the principle here was much more complicated and that in general the action of any electrode in the polyelectrode system depended not only upon the two electrodes with the most widely differing potentials in the given system, but also upon the effective potentials of all the other electrodes, upon the relative surface area of each electrode and upon the ohmic resistance in its different circuit. While such a general solution of this problem gives the most correct explanation of the mechanism of this phenomenon, it does not offer any possibility of making a quantitative calculation, if the system consists of more than three electrodes.

The new line which has been taking root of late, especially after a number of papers by Evans ([3]), Brown and Mears ([4]), Wagner and Traud ([5]), Frumkin([6]) in the study of corrosion, is to compare the cathode and the anode polarization

curves. With this the problem of a polyelectrode element may be simplified considerably, and a suitable method can be evolved calculating quantitatively the work of each separate electrode in a polyelectrode system with any number of electrodes.

In order to make the calculation, the relative surface area of each electrode in the system must be known, and the cathode and anode polarization curves (current density—potential) under conditions close to the working conditions of the whole system must be plotted beforehand for each component of the system separately. This method of calculation is based upon

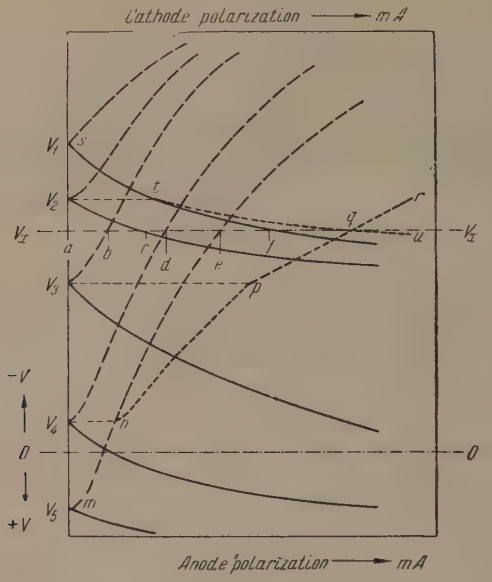

Fig. 1. Curves of anode polarization (full lines); curves of cathode polarization (dotted lines); *stqu*—summary cathode polarization curve; *mnpqr*—summary anode polarization curve.

the assumption that in an acting polyelectrode system the effective potentials of all the electrodes of the system become equal as a result of polarization. This is generally the case if the total ohmic resistance is small (e. g. if the separate components of the system are very close to one another, or if the conductivity of the electrolyte is high enough).

According to the area of each electrode, we replot the curves c u r r e n t d e n s i t y-p o t e n t i a l mentioned above into a general c u r r e n t-p o t e n t i a l diagram.

Fig. 1 gives such a diagram schematically for a five-electrode system. Here V_1, V_2, V_3, V_4, V_5 are the potentials of the separate components of the system (electrodes *1*, *2*, *3*, *4* and *5*, respectively) without current. The anode polarization curves are represented by full lines and the cathode polarization curves by dotted lines. This diagram gives the current values for each electrode of the system in relation to the general potential in the system. This general potential of a polyelectrode system (V_x) equals the potential, at which the sum of all the cathode currents in the system equals the sum of all the anode currents, i. e. is determined by the point of intersection (*q*) of the summary c u r r e n t-p o t e n t i a l curve for all the cathode processes and the summary curve of all the anode processes. The summary

curves of anode (*stqu*) and cathode (*mnpqr*) polarization are produced by simply adding the currents of all the anode processes for each potential between V_1 and V_5 and, respectively, for all the cathode processes within the same potential interval.

The intercepts measured along the abscissa axis from the origin to the point, at which the cathode or anode polarization curves, respectively, cross the abscissa V_x, give the direction and magnitude of the current in each electrode of the system. In Fig. 1 the abscissa of V_x is intersected by the cathode polarization curves of electrodes *3*, *4* and *5*, therefore these electrodes will act as cathodes and their cathode currents will be represented by the intercepts: *ab*, *ad* and *ae*, respectively.

For electrodes *1* and *2* the abscissa V_x is intersected by their anode polarization curves. Therefore in this system electrodes *1* and *2* will act as anodes, whose anode currents may be determined from the intercepts *ac* and *af*, respectively.

The diagram shows that the anode polarization curve for the most positive electrode and the cathode polarization curve for the most negative electrode are superfluous in this plot, since they cannot be summed up with the curves of anode and cathode polarization, respectively, within the potential interval between V_1 and V_5.

As it is, the electrodes with intermediate potentials require also only one curve—either the anode polarization curve or the cathode polarization curve; but before the general diagram has been constructed, it cannot possibly be predicted wether the cathode or the anode curves will make up the sum, therefore in general it is necessary to construct both the cathode and the anode curves.

Sometimes the polarization curves necessary for the calculation of polyelectrode systems can be obtained without experiment directly from the theoretical curves of elementary cathode and anode processes by means of calculation. And so they have been obtained by Frumkin ([6]) for Na-amalgam and Pb; by Wagner and Traud ([5]) for Zn-amalgam. However, this is possible for the time being in a very limited number of cases.

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CHEMISTRY

TESTS OF POLYMERIZATION UNDER PRESSURE

By O. I. LEIPUNSKY and N. M. REINOV

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In testing apparatus for studying gases (in small quantities) at high pressure several experiments on polymerization were performed. They were not (and will not be) developed further and have remained roughly qualitative tests. However, in view of some interest to these problems, we resolve to make a report of our work in this short paper.

Experiments were performed with 3 substances—butylene, formaldehyde and acetaldehyde at 5000 to 6000 atm. The temperature was 280 to 320° C, duration 30 min. The substance was put into a capillary of stainless steel, the pressure was transmitted by mercury; $^2/_3$ of the volume of the capillary filled with the substance was heated. The method is described more elaborately in ([1]) and ([2]). The products were not analyzed chemically for lack of materials. Polymerization was not observed in formaldehyde. The pulverized formaldehyde decomposed completely liberating small quantities of carbon. Acetaldehyde on the contrary polymerized easily. Fowcett ([3]) produced a viscous liquid with a large quantity of volatile fractions at 3000 atm., 150° in 16 hrs. The residue after distillation was a rubbary mass.

We obtained a red brown mass, apparently similar to Fowcett's residue. Butylene also polymerized easily. At 5000 atm. it gave a liquid polymer. We used butylene with a content of not over 5% admixtures (mainly pseudo- and isobutylene). This polymer is an oily liquid (viscosity 1.5—2 times that of water), pale yellow in colour. The boiling of most of the polymer is roughly estimated (by comparing the evaporation of the polymer with the evaporation of alcohol and water) at 200° C.

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PHYSICAL CHEMISTRY

SETSCHENOFF'S RULE AND THE SOLUBILITY OF HYDROGEN SULPHIDE IN HYDROCHLORIC ACID SOLUTIONS

By A. F. KAPUSTINSKY, Corresponding Member of the Academy, and B. I. ANVAER

Setschenoff ([1]) found the following empiric formula, relating the solubility of gas in a solution of the electrolyte to the concentration of the latter:

$$s = s_0 e^{-kc} \qquad (1)$$

where s_0 is the solubility of gas in water; s, the solubility of gas in a solution of the electrolyte; c, the concentration of the electrolyte.

Randall and Failey ([2]) interpreted this formula in a more modern form, representing it (cf. equation 2) as a proportionality between the logarithm of the activity coefficient (γ) of the non-electrolyte and the ionic strength of the electrolyte (μ). As the ionic strength for the electrolytes of the same type of valency is proportional to the concentration, while the activity coefficient of the dissolved substance is determined as a ratio of the gas concentration in water to the gas concentration in a salt solution, for salts of the same type of valency Randall and Failey's formula is practically equivalent to that of Setschenoff. The novelty of their work consisted in establishing the approximate invariability of the constant of salting out for different gases and non-electrolytes with the same electrolyte.

Setschenoff's formula requires a monotonous change of gas solubility with the electrolyte concentration. On the other hand, Kendall and Andrews ([3]), having measured the solubility of hydrogen sulphide in hydrochloric acid up to a concentration of 5 mols per liter, found that the solubility of H_2S passes through a minimum. According to Geffcken ([4]), the addition of small amounts of hydrochloric acid reduces the solubility of the gases (H_2, O_2, CO_2, N_2O) in water, while from other studies it is known that the solubility of gases in concentrated hydrochloric acid is higher than in water [cf. Manchot's ([5]) data for nitrous oxide, Hoffmann's ([6]) for carbon dioxide, and those of Sherrill and Izard ([7]) for chlorine]. Therefore it is of interest to check the applicability of Setschenoff's formula to the solubility of H_2S in HCl throughout the concentration range.

In the present study measurements have been made of the solubility of H_2S in hydrochloric acid of higher concentration than that used in the work of Kendall and Andrews; the effect of $FeCl_2$ and $ZnCl_2$ upon the solubility of H_2S in hydrochloric acid has also been investigated.

Method of Measurements. The determinations were made at a temperature of $25 \pm 0.1°$ C. The hydrogen sulphide which we obtained in the same way as Scheffer ([8]) and Bell ([9]), from chemically pure hydrochlo-

ric acid (1 : 1) and a concentrated solution of NaHS, and which was purified from CO_2 by means of $Ba(OH)_2$, was passing through a vessel placed in a thermostat; in this vessel was placed a portion of the solution under investigation in order that the gas be saturated with the vapour of the solution studied. Hence, through a pipe reaching the bottom of the vessel and supplied with a number of small perforations, it passed into a vessel for absorption with a capacity of 300 ml placed also in the thermostat. The pipe with a stop-cock placed near the bottom of the vessel permitted to take a sample of the solution for analysis.

The vessel was made to contain from 100 to 150 ml of a solution of chemically pure hydrochloric acid of a given concentration. As a rule, the saturation was attained within 3—4 hours. The content of hydrogen sulphide of the sample was determined iodometrically. In order to prevent the oxidation of H_2S by the oxygen of the air, we thoroughly washed the apparatus with nitrogen prior to passing the hydrogen sulphide.

The HCl content in the solution was determined after the experiments, according to Vollhard. In our calculations we made use of the values of the vapour pressures of hydrochloric acid of various dilutions, obtained by Zeisberg ([10]) and Jannakis ([11]). The solubility in grammols per liter of solution, corresponding to a partial pressure of hydrosulphuric acid of 760 mm, was obtained from the data of analysis by introducing corrections for barometer pressure, vapour pressure of the solution and the specific gravity of the solution, which we determined with the aid of an areometer with an accuracy to within 0.2%.

According to our measurements, the solubility of hydrogen sulphide in water equals 0.1013, while, according to the data of Kiss, Lajtaj and Thury ([12]), it equals 0.1014; according to those of Wright and Maass ([13]), 0.1019; of Winkler ([14]), 0.1018; of Kendall and Andrews ([3]), 0.1023; and of Pollitzer ([15]), 0.1004; in a 3 *N* NaCl solution, 0.0548, while after Irk ([16]) it is 0.0540; and, finally, for a 1 *N* KCl solution, 0.0877, while McLauchlan ([17]) obtained 0.0873. Thus, the accuracy of our measurements is quite satisfactory.

In order to determine the solubility of hydrosulphuric acid in a hydrochloric acid solution of zinc chloride, we used the same method. Acid solutions of zinc chloride were prepared by dissolving chemically pure granulated zinc in hydrochloric acid. $FeCl_2$ being readily oxidized, its acid solutions were prepared by

Table 1

Hydrogen chloride			Hydrogen sulphide			
			Under partial pressure of 760 mm			
			grammols per liter of solution	grammols per 1000 g of water		Under total pressure 760 mm
weight %	grammols per liter of solution	grammols per 1000 g of water		found	calculated after Setschenoff	
0	0	0	0.1023	0.1026	0.103	0.0993
0.49	1.135	0.135	0.1018	0.1025	0.103	0.0993
1.02	0.283	0.285	0.1015	0.1025	0.104	0.0993
2.28	0.631	0.64	0.1016	0.1032	0.105	0.1000
4.22	1.18	1.21	0.1020	0.1048	0.106	0.1018
6.54	1.85	1.91	0.1026	0.1067	0.108	0.1037
8.74	2.50	2.62	0.1030	0.1088	0.110	0.1059
10.55	3.04	3.23	0.1034	0.1105	0.113	0.1076
11.7	3.31	3.53	0.1039	0.1119	0.114	0.1091
15.0	4.41	4.82	0.1076	0.1186	0.118	0.1159
16.5	4.87	5.39	0.1090	0.1215	0.121	0.1188
16.8	5.0	5.55	0.1098	0.1227	0.121	0.1200
20.2	6.1	7.0	0.1124	0.1291	0.125	0.1266
23.7	7.3	8.4	0.1165	0.1375	0.132	0.1351
28.1	8.8	10.7	0.1192	0.1462	0.142	0.1437
34.9	11.4	14.7	0.1169	0.1542	0.158	0.1319

Solubility of H_2S in HCl for concentrations of hydrochloric acid from 0 to 16.5% has been calculated from data of Kendall and Andrews; from 16.5 to 34.9%, measured by us.

dissolving a preliminarily analyzed sample of Kahlbaum ferrous sulphide in hydrochloric acid of a known concentration. Shortly before the full dissolution of iron sulphide, the flask was closed with a stopper with a pipe supplied with a glass filter, and the hydrogen sulphide evolved was forcing out the solution through the pipe into the weighed flask with a titrated iodine solution. Experiments have shown that sampling, before a complete dissolution of FeS has taken place, may lead to an error of 1—1.5%.

Results of Measurements. The solubility of hydrogen sulphide in hydrochloric acid solutions is given in Table 1, and the solubility of hydrogen sulphide in hydrochloric acid containing zinc chloride and ferrous chloride, in Tables 2 and 3.

Since there are no data upon the vapour pressure of H_2O and HCl over hydrochloric acid solutions of ferrous chloride and zinc chloride, for these solutions we cite values of hydrogen sulphide solubulity under a total pressure equal to the atmospheric one.

From the data obtained (Table 1) it is seen that when recalculating the concentration in the form of molality, the solubility has no minimum. Within the entire range of concentrations the solubility of hydrogen sulphide in hydrochloric acid is satisfactorily expressed by Setschenoff formula with an average coefficient of salting-out equal to—0.013.

$$\frac{\lg\gamma}{\mu} = -0.013. \qquad (2)$$

As seen from column 6 of Table 1, the deviations do not exceed 4%.

Table 2

Solubility of Hydrogen Sulphide in Solutions of $HCl—ZnCl_2—H_2O$

Per 1000 g of water grammols of			Total pressure in mm	$\lg\gamma/\mu$
HCl	$ZnCl_2$	H_2S		
3.9	2.1	0.1157	760	—0.0029
5.0	4.7	0.1501	758	—0.0070
7.8	1.0	0.1345	755	—0.0038
9.1	1.5	0.1433	746	—0.0039
10.6	1.1	0.1495	755	—0.0061
13.5	0.8	0.1416	750	—

Table 3

Solubility of Hydrogen Sulphide in Solutions of $HCl—FeCl_2—H_2O$ (under a total pressure equal to 1 atm.)

Per 1000 g of water grammols of			$\lg\gamma/\mu$
HCl	$FeCl_2$	H_2S	
1.6	0.9	0.085	0.031
3.4	1.0	0.089	0.028
7.8	1.1	0.108	0.026
10.6	1.2	0.120	0.023
9	saturated (less than 20%)	0.101	—

It is of interest to trace the course of solubility in the region of still higher concentrations of hydrogen chloride than those with which we worked. No similar experiments could be performed with our apparatus, since they require an additional saturation of the solution with hydrogen chloride. An extrapolation of the data obtained is unreliable. However, there is a possibility to settle this question in the following way. If the solubility of H_2S be plotted on a graph under a total pressure of 760 mm, the curve must upon continuation reach a point lying on the axis of concentrations of hydrogen chloride and corresponding to the composition of a solution of HCl boiling at 25°, or, in other words, to the solubility of hydrogen chloride in water at 25° under a total pressure of 760 mm.

Between the point of solubility of H_2S in hydrochloric acid of the highest concentration studied by us, and the point of solubility of HCl in water an approximate interpolation may be made, and in this way an estimation of the value of solubility of hydrogen sulphide in hydrochloric acid may be obtained for concentrations of HCl from 15 to 18 grammols per 1000 g of wa-

ter, i. e. from 36 to 40%, under a total pressure of 760 mm. Hence the values of solubility of hydrogen sulphide under a partial pressure of 760 mm may be calculated (Table 4). It is found that the solubility of hydrogen sulphide under a partial pressure of 760 mm in the above range of concentrations of HCl continues to grow in accordance with Setschenoff's formula.

Table 4

HCl		Solubility of H_2S in grammols per 1000 g of water		
			under a partial pressure of 760 mm	
weight %	grammols per 1000 g of water	under a total pressure of 760 mm (according to the graph)	according to the graph	according to Setschenoff's formula
34.9	14.7	0.132	0.154	0.158
36.5	15.7	0.125	0.16	0.164
37.7	16.6	0.11	0.165	0.168
39.0	17.5	0.08	0.17	0.173
40.3	18.5	0.045	0.175	0.178
41.5	19.4	0.00	0.18	0.183

From the data found by us for the solubility of H_2S in acid solutions of $FeCl_2$ and $ZnCl_2$, it is possible to calculate the constants of salting-out of these salts. Assuming approximately that the salting-out action of the salt does not depend on the concentration of the acid present and comparing the solubility of hydrogen sulphide at the same concentration of HCl in the presence and in the absence of the salt, we obtain:

$$\text{for } FeCl_2 \text{ (Table 3) } \lg\gamma/\mu = 0.027 \pm 0.004$$

$$\text{» } ZnCl_2 \text{ (» 2) } \lg\gamma/\mu = -0.005 \pm 0.002.$$

The constant for the high concentration of $ZnCl_2$ is near the value calculated from the data of Kobe and Williams ([18]) on the solubility of CO_2. From their data for a 50% solution of $ZnCl_2$ (7.3 grammols per 1000 g of H_2O) we obtain $\lg \gamma/\mu = 0.0076$. Thus, $ZnCl_2$, as well as HCl, does unquestionably increase the solubility of gases in water.

Conclusions. 1) A study of the solubility of hydrogen sulphide in hydrochloric acid of different concentrations has shown that the Setschenoff-Randall-Failey rule is applicable to the above system, and that hydrochloric acid increases the solubility of hydrogen sulphide in water throughout the range of concentrations. 2) A study of the effect of $ZnCl_2$ and $FeCl_2$ upon the solubility of H_2S in HCl has shown that $ZnCl_2$ increases, and $FeCl_2$ reduces the solubility of H_2S in HCl.

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PHYSICAL CHEMISTRY

ON THE HEAT CONDUCTION OF SOME EXPLOSIVES

By A. BELAJEV and N. MATUSHKO

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For the development of a complete theory of explosives, especially treating the problems of their ignition, combustion, and detonation, the knowledge of at least the approximate value of thermal conductivity for typical explosives is necessary; the present work aims to solve this problem. We have determined experimentally the approximate value of the temperature conductivity a for some explosives. If the density of the substance γ and the specific heat c are known, the thermal conductivity coefficient λ can be calculated by means of the equation

$$\lambda = a\gamma c.$$

The method adopted by us was based on the following principles: let us suppose that a substance with a temperature T_0 is placed in close contact with a good heat conductor with a temperature T_1. In the substance, at a distance S from its surface, is placed a thermocouple. Suppose that after a time τ following the contact the thermocouple shows the temperature T'. From the heat theory it follows that a definite relation exists between the values of S, τ and the temperature conductivity a; namely, if the ratio of the temperature differences $\frac{T_1 - T'}{T_1 - T_0}$ remains constant for a series of experiments, then $\frac{a\tau}{S^2} = \text{const}$. We took a substance, for which the temperature conductivity constant a was known and, maintaining constant the ratio $\frac{T_1 - T'}{T_1 - T_0}$ (experimentally this was obtained by a previous calculation of T' from the values of T_0 and T_1, and τ was recorded at the moment T' was attained), we determined the value of $\text{const} = \frac{a\tau}{S^2}$. Then, under the same conditions, τ was determined for a substance with an unknown temperature conductivity a, that was afterwards calculated from the expression:

$$a = \frac{\text{const}\, S^2}{\tau}.$$

As primary substances we used firwood, quartz sand and, sometimes, cork. We determined the temperature conductivity for explosives in different forms: as tablets compressed at 1000—2000 atm., in the form of powder and in liquid state. In the case of solids we heated the substances from the bottom, in the case of liquids we cooled them from the bottom to reduce convection. T_0 in our experiments was about 20° C, T_1 about 70° C.

In the case of liquid and powdered explosives the thermocouple was placed beforehand at a definite distance from the heating surface. In the case of compressed powder explosives the thermocouple was placed between two tablets, the distance S being fixed according to the thickness of the tablet. The data obtained by us are tabulated below: γ is the density of the substance, as determined by us; c, its specific heat (the values with asterisks are taken from other authors, the rest we determined approximately); a, its temperature conductivity, and λ, its thermal conductivity.

Substances	γ, g/cm³	c, cal./deg. g	$a \times 10^4$ cm²/sec.	$\lambda \times 10^4$, cal./cm sec. deg.
Explosives in tablets:				
1. Trotyl	1.56	0.36*	8.5	4.8
2. Tetryl	1.7	0.225*	6.0	2.3
3. Picric acid	1.60	0.26*	5.8	2.4
4. Pyroxyline	1.50	0.18	8.0	2.15
5. Dinitrotoluene	1.50	—	5.0	—
6. Dinitronaphthalene	1.50	—	12.7	—
7. Lead azide[1]	4.1	0.09	11.0	4.2
8. Mercury fulminate	3.8	0.1	7.5	2.85
Powdered explosives:				
1. Trotyl	0.846	0.36*	11.5	3.5
2. Tetryl	0.767	0.225*	11.5	2.0
3. Lead azide	0.88	0.09	19.5	1.55
4. Mercury fulminate	0.81	0.1	13.6	1.1
Liquid and gelatinised explosives:				
1. Nitroglycol	1.5	0.4	(14.0)	(8.4)
2. Gelatinised nitroglycol	~1.5	~0.4	8.5	5.1
3. Nitroglycerine	—	—	(10.5)	—
4. Gelatinised nitroglycerine	—	—	8.0	—

[1] To obtain solid tablets we added to the lead azide a small quantity of paraffin.

It should be pointed out that the above data cannot claim to considerable accuracy, especially, if absolute values are needed; besides, all the measurements were made at temperatures little different from room temperatures. The considerable effect of gelatinisation (accomplished by ~5% of pyroxyline) upon the value of thermal conductivity is due to the insufficience of the precautions we took against convection. The respective values of a and λ are taken in parantheses. From the data given in the table one can note that for nitroglycerine, evidently because of its greater viscosity, the effect of convection is smaller. Let us draw some conclusions from the above data. It is known that mercury fulminate, all other conditions being equal, is much more sensitive to a fire beam than lead azide. It is also known that in powdered state these explosives have a higher sensitivity to fire than in pressed powder state. We will show that these facts can be easily explained by means of the above data. Let us supply to the surface of the body a quantity of heat q cal. per sec. Then the temperature rise on the surface will be:

$$T' - T_0 \cong k \frac{q\sqrt{\tau}}{\sqrt{\lambda c \gamma}};$$

consequently, if we supply heat at such a velocity that the temperature on the surface of powder mercury fulminate will attain the ignition point (170° C) after a time τ, then after the same time τ, the temperature on the surface of powdered lead azide will be 150° C (ignition point —330° C), on the surface of mercury fulminate in compressed state 65° C and on the sur-

face of lead azide in pressed powder state 55° C. If we apply heat upon the surface of mercury fulminate in pressed powder state at such a rate, that the temperature upon its surface will attain its ignition point in one second, then the time necessary to attain the ignition will be respectively: $\sim$0.1 sec. for powdered mercury fulminate, $\sim$0.5 sec. for powdered lead azide and 7 sec. for lead azide in pressed powder state (of course, all these calculations concern the densities given in the table). Thus, in this case, the different inflammabilities of fulminating mercury and lead azide in different states can be explained by the heat theory, and there is no need in other suppositions.

We will point out that even before this work was commenced, Chariton and Rdultovskaya, in studying the thermal explosion of lead azide, concluded that it was sufficiently well descrided by the above method. It is known that gelatinised nitroglycol and nitroglycerine are ignited with less difficulty than ungelatinised. Evidently, this can be explained by the fact that gelatinisation hampers the heat transfer by convection that occurs in liquid substances. As shown by the table, convection plays a certain part, but the difference in heat transfer for gelatinised and liquid substances is not large enough to explain the marked difference in the inflammabilities of these two types of substances. It is evident that either the convection during ignition is more intense than under our experimental conditions, or the easy inflammability of gelatinised explosives in comparison with the same substances in liquid state is explained by some other reason.

Let us note another fact. For substances like trotyl, picric acid, tetryl the temperature conductivity values were measured at relatively low temperatures. If heated, these substances will melt long before their ignition point is attained. This change from a solid to a liquid state will naturally bring forth a slowing down of the further heating of the surface of the explosives. Contrarily, in the case of pyroxyline that does not pass into liquid state, such a slowing down can be absent even for temperatures close to its ignition points; in this way can be explained the easy inflammability of pyroxyline, even in compressed powder state, as compared with other secondary explosives.

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PHYSICAL CHEMISTRY

ON THE IGNITION OF SECONDARY EXPLOSIVES

By A. BELAJEV and E. SAMBURSKAYA

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The present work aims to ascertain the basic factors that determine the degree of ignitibility of secondary explosives.

In a paper by one of the authors (1), it is stated, that the combustion of explosives proceeds in the gas phase. From this it immediately follows that the temperature of the surface of burning explosive must be equal to its boiling point. It means, that for igniting a given explosive, one must first of all create on its surface a temperature, which approaches its boiling point. Thus, we come to the conclusion, that the inflammability of secondary explosives must be a function of their volatility. In conducting this work it was necessary above all to adopt a definite method of a relative estimation of inflammability. The ignition of explosives is considerably facilitated by the increase of temperature. This fact can be taken as a basis for a relative estimation of inflammability—«sensitivity» to ignition. In our experiments we ignited the explosives by means of a nichrome wire 0.4 mm in diameter, heated to cherry red incandescence by electric current. The current and the incandescence were kept constant during all the experiments. The substance to be ignited was introduced in a glass placed on an electrical heater. The igniting wire touched the surface of the explosive. The temperature of the explosive was measured just before the inflammation by means of a low-inertia thermocouple, placed in the substance. The current was switched on for 5 sec. each time. If it did not result in a total combustion, the substance was heated by 5—10° C and the current was switched on again for 5 sec. Successive heatings were performed until at a certain temperature total inflammation and combustion took place. This temperature, at which our «heat impulse» resulted in inflammation, can be taken as a relative standard of inflammability. The lower this temperature, the easier is the inflammation of the explosive, the more sensitive it is to ignition. In another series of experiments we used a thin gas flame applied for one second. The experimental data are given in the table. The explosives are classed according to their degree of inflammability.

T' is the temperature, at which the explosive was ignited by means of the wire; T'', the temperature of ignition by means of the gas flame; T_k, the boiling point of the substance (2). It is clear, that the temperature values given in the table can only be used as a relative standard of inflammability. Even at temperatures 10—20° C below total ignition, for most substances are observed flashes that bring forth no total combustion. Nearly for all substances the combustion proceeds calmly at small velocities (from 0.1 to

1 mm/sec.), except trinitroglycerine that gives at 140° C a vigorous flash with a clap, partly throwing out the burning substance, while the glass remains intact. Indeed, a glance at the table is sufficient to see, that inflammability is in the first place a function of the boiling point. As a rule, the higher the boiling point of an explosive, the more difficult is its ignition. The difference between the boiling point and the temperature of ignition by a standard igniter is for most substances of the same magnitude. Some deviations from this rule, that are especially noticeable in the case of hexogen and nitropentaerythrite, can be explained by the influence of other physical factors. Thus, hexogen by its inflammability is similar to nitroglycol, while its boiling point is higher than that of tetryl. The relatively easy inflammability of hexogen can be explained by its high melting point, that must influence the heat conduction conditions of the substance.

Substance	T', °C	T'', °C	T_k, °C
Methyl-nitrate	−50	−75	65
Hexogen	—	70	275
Nitroglycol	105	80	200
Trinitroglycerine	140	—	250
Tetryl	—	135	255
Nitropentaerythrite	155	—	225
Trinitrobenzene	195	—	300
Trinitrocresol	210	180	305
Trotyl	225	190	310
Trinitroanisol	225	195	310
Picric acid	—	200	325
Dinitronaphthalene	235	220	300

If the inflammability of an explosive is a function of its boiling point, then the inflammability must be facilitated by reducing the pressure, because the boiling point will decrease with the reduction of the pressure and the relative vapour concentration in the air-gas mixture will increase (it is clear that, in the case of a very strong reduction of pressure, combustion in the gas-phase can be made utterly impossible). It was shown in our experiments that nitroglycol is ignited at a pressure of 200 mm Hg by means of the wire at a temperature that is by 15—20° C below its ignition temperature at 760 mm Hg; correspondingly, trotyl at a pressure of 350 mm Hg is ignited at a temperature that is by 10—15° C below its ignition temperature at a pressure of 760 mm Hg. Especially interesting results, while reducing the pressure, were obtained with nitroglycerine. In a paper by one of the authors with Jusephovich [3] it was pointed out that the boiling point of nitroglycerine is much higher than the temperature that corresponds to a considerable development of the reaction, and this very fact was supposed to determine some peculiarities in the behaviour of nitroglycerine during its ignition. At pressures about 100 mm Hg nitroglycerine boils at a temperature close to 200° C (the boiling point of nitroglycerine at 760 mm Hg is about 250° C). Consequently, at this pressure the boiling point of nitroglycerine is equal to that of nitroglycol at atmospheric pressure. Being ignited at a pressure of 100 mm Hg by a standard igniter, nitroglycerine inflamed at a temperature about 110° C, i. e. at the same temperature, at which nitroglycol inflames at atmospheric pressure. Further, at a pressure of 100 mm Hg, quite contrary to the process at atmospheric pressure, the combustion of nitroglycerine is stable* and is approximately similar to that of most of the secondary explosives after their inflammation. Consequently, both in regard of inflammability and of the character of combustion (except the velocity of combustion) nitroglycerine at 100 mm Hg is similar to nitroglycol

* The stable combustion of nitroglycerine at reduced pressures was observed by Andrejev [4], but his explanation of this phenomenon does not coincide with our explanation.

at atmospheric pressure, which fully corroborates the above statement concerning the reasons of the peculiar behaviour of nitroglycerine while ignited at atmospheric pressure.

All the experimental data obtained by us claim that the inflammability of most secondary explosives is a function of their boiling point. In the case of involatile substances the ignition facility seems to be determined by the self-inflammation point and the heat conduction of the given substance. In conclusion, it should be pointed out, that pyroxyline proves to be a striking exception among secondary explosives—it inflames without difficulty and burns at low temperatures, although it does not possess appreciable volatility. The problem of ignition and combustion of pyroxyline stands in need of an independent treatment.

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1941. Volume XXX, № 7

PHYSIKALISCHE CHEMIE

ÜBER DIE PHOTOCHEMISCHE UNTERSUCHUNG DES DUNKLEN ANILINS

Von A. T. WARTANJAN

(Vorgelegt von A. N. Terenin, Mitglied der Akademie, den 23. XI. 1940)

Es ist unmöglich auf Grund der in der Literatur vorhandenen Angaben ([1], [2], [3]) die Frage zu beantworten, ob das Dunkelwerden des Anilins von diesem selbst oder von anderen Verbindungen herrührt.

Um diese Frage zu erläutern, untersuchten wir das Verhalten von chemisch reinem Anilin im Hochvakuum (10^{-5} mm). In ein evakuiertes und sorgfältig entgastes Quarzgefäss wurde mehrmals an der Luft und zuletzt im Vakuum destilliertes Anilin eingeschmolzen. Wie die Versuche zeigten, bleibt Anilin, sowie im Dunkeln, als auch im zerstreuten Tageslicht während unbestimmt langer Zeit vollständig farblos. Dasselbe Anilin verfärbt sich jedoch intensiv bei der Belichtung mit Magnesiumfunken, deren Licht entweder durch isländischen Spat oder Gelatinefilm* filtriert wird und erinnert hierbei an gewöhnliches dunkelgewordenes Anilin. Bei Temperatursteigerung wächst rasch die Geschwindigkeit der photochemischen Reaktion, wobei sich die Wände des Quarzgefässes mit schwerflüchtigen Verharzungsprodukten bedecken. In einer Arbeit über den photothermischen Zerfall des Anilins ([4]) konnten wir zeigen, dass das thermische Auslöschen der Fluoreszenz des Anilins durch photochemische Veränderung des Anilinmoleküls bedingt wird. Somit erleidet Anilin im ultravioletten Lichte ($\lambda\lambda \leqslant 2800$ Å) photochemische Umwandlungen.

Im Reaktiongefäss konnte die Anwesenheit von Wasserstoff, Ammoniak und einer gefärbten Substanz (oder möglich eines Gemisches gefärbter Substanzen) festgestellt werden. Entgegen den Behauptungen von Gibbs ist es uns nicht gelungen spektroskopisch die Anwesenheit von Benzol festzustellen. Diese Tatsache beruht auf der auslöschenden Wirkung des Anilins, der zufolge die Stokesche sensibilisierte Fluoreszenz des Anilins in Benzoldämpfen beobachtet wird, d. h. folgender Prozess: Anilin + Benzol* → Anilin* + Benzol.

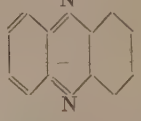
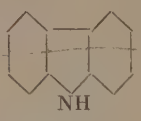
Nach Prilejaewa ([5]), die die Existenz des oben angeführten Prozesses experimentell nachweisen konnte, verläuft die Energieübergabe mit so hoher Wahrscheinlichkeit, dass praktisch jeder Zusammenstoss als effektiv gerechnet werden kann. Deshalb muss die Fluoreszenz einer geringen Menge von Benzoldämpfen bei Anwesenheit eines Überschusses an Anilindämpfen vollständig ausgelöscht werden.

Bei Anregung der Dämpfe des im Quarzgefäss entstehenden Reaktionsgemisches, wird eine intensive gelb-grüne Fluoreszenz mit einem kontinuier-

* Isländischer Spat absorbiert das ultraviolette Licht von 2100 Å, Gelatinefilm — von 2450 Å an.

lichen Spektrum im Gebiete von 6300—3840 Å (Abb. 1, *a*) beobachtet (4). Zur Identifizierung des Trägers des gelb-grünen Leuchtens betrachten wir

Hydrazobenzol ($C_6H_5NH \cdot NHC_6H_5$)

Phenazin  , Carbazol  und

Azophenin ($C_6H_2(NC_6H_5)_2(NHC_6H_5)_2$)*

Ein intensives gelb-grünes Leuchten wird bei 400° C beobachtet. Bei dieser Temperatur kann man keine Fluoreszenz der Hydrazobenzoldämpfe beobachten, da dieses zersetzt wird und in Azobenzol übergeht. Marsh hat jedoch eine Tesla-Luminiszenz der Hydrobenzoldämpfe ($p \sim 1—2$ mm) in Gestalt eines schwachen kontinuierlichen Bandes im Gebiet von 4125—3050 Å mit einem Maximum für $\lambda = 3340$ Å, beobachtet (8). Es ist klar, dass Hydrazobenzol nicht der Träger des gelb-grünen Leuchtens sein kann.

Ausser Isobenzidin gelang es Bernthsen eine geringe Menge Phenazin zu isolieren (9). Die photochemische Bildung von Phenazin aus Anilin wurde von Malaviya und Datt beobachtet (10). Die Phenazinkristalle zeigen bei Zimmertemperatur eine intensive gelb-grüne Fluoreszenz; die durch filtriertes Licht des Quecksilberbogens angeregten Dämpfe jedoch besitzen eine sehr schwache grüne Fluoreszenz**. Bei 400° C erleidet Phenazin einen photothermischen Zerfall, wonach das Reaktionsgemisch eine intensive orangegelbe Fluoreszenz besitzt.

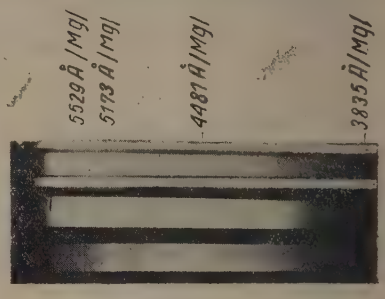


Abb. 1.

Man konnte annehmen, dass das intensive gelb-grüne Leuchten der Dämpfe des Reaktionsgemisches entweder durch die sensibilisierte Fluoreszenz des Phenazins in Anilindämpfen oder durch die stabilisierende Wirkung der Anilinmoleküle hervorgerufen wird. Wie die Versuche zeigten, leuchtet das Gemisch von Phenazin- und Anilindämpfen genau so schwach, wie die reinen Dämpfe des Phenazins. Wird also infolge photochemischer Reaktionen Phenazin gebildet, so sind weder Phenazin, noch die Produkte seines photothermischen Zerfalls die Träger des gelb-grünen Leuchtens.

Unter den Produkten des pyrolytischen Zerfalls von Anilin konnte Hofmann (11) eine geringe Menge einer kristallinen Substanz auffinden, die nach Graebe (12) Carbazol darstellt. Festes Carbazol besitzt eine blau-violette Fluoreszenz (13). Wie unsere Versuche zeigten, fluoreszieren die durch Quecksilberbogenlicht oder Zink- bzw. Magnesiumfunken angeregten Carbazoldämpfe im Gebiet $\lambda\lambda = 4900—3280$ Å. Folglich, ist Carbazol ebenfalls nicht der Träger des gelb-grünen Leuchtens.

Zur Prüfung des Azophenins mussten wir einen Umweg machen, da wir

---

* Benzidin (H_2N – – – NH_2) wird hiervon ausgeschlossen, da das Fluoreszenzspektrum der Benzidindämpfe im Gebiete $\lambda\lambda = 4500—2950$ Å (6) liegt. Beim Durchleiten von Anilindämpfen durch ein rotglühendes Rohr konnte Bernthsen (7) unter den Zerfallsprodukten eine äusserlich dem Benzidin ähnliche Substanz feststellen, von gleicher chemischer Zusammensetzung ($C_{12}H_{12}N_2$). Jedoch unterscheidet sich diese Substanz in ihren Eigenschaften merklich vom Benzidin und wird Isobenzidin genannt. Über die Emission des Isobenzidins ist nichts bekannt.

** Unter der Wirkung von Röntgenstrahlen zeigt Phenazin eine intensive blau-grüne Fluoreszenz ($\lambda\lambda = 560—400$ mμ) (22).

über die reine Substanz nicht verfügten. Wenn für das gelb-grüne Leuchten Azophenin verantwortlich gemacht werden kann, so muss dieses Leuchten in den Dämpfen des gewöhnlichen dunklen Anilins, sowie in den Dämpfen von dessen festen Rückständen beobachtet werden. Das Fluoreszenzspektrum des dunklen Anilins ist in Abb. 1, *b* wiedergegeben. Wie die Versuche zeigten, bestehen die festen Rückstände des dunklen Anilins aus einem flüchtigen und einem nicht flüchtigen Anteil. Der flüchtige Anteil stellt eine Substanz von roter Farbe dar; diese wurde in ein evakuiertes Glasgefäss überdestilliert. Das Fluoreszenzspektrum dieser Substanz ist in Abb. 1, *c* wiedergegeben. Ein Vergleich der drei Spektra zeigt, dass diese im wesentlichen identisch sind. Wesentlich ist, dass der in Abwesenheit oxydierender Substanzen unter der Wirkung des ultravioletten Lichtes entstehende Träger des gelb-grünen Leuchtens an der Luft unter der Wirkung des Tageslichtes gebildet wird. Eine derartige Bildung muss einer autokatalytischen Wirkung zugeschrieben werden. Es ist möglich, dass das Azophenin im wesentlichen der Träger des gelb-grünen Leuchtens ist. Hierbei ist es nicht ausgeschlossen, dass auch einige andere Zerfallsprodukte, wie z. B. Phenazin und die Produkte seines photothermischen Zerfalls, leuchten können und ihre Spektra werden das Grundleuchten des Azophenins überlagern.

Primäre Vorgänge sind die Abspaltung der Aminogruppe und des Wasserstoffatoms vom Phenylrest. Die Realität dieser Vorgänge wird durch die Analyse des Absorptionsspektrums von Anilin ([14–18]) überzeugend. Die Absorption, die der Aminogruppe zugeschrieben wird, beginnt etwa bei 2420 Å und verbreitet sich in Richtung der kurzen Wellen. Im Gebiet 2420—2033 Å wird eine diskontinuierliche Absorption, die in 3 Serien zerfällt, beobachtet. Diese Serien von Banden werden als nachfolgende Banden gedeutet, die infolge von Übergängen von dem Niveau $v''=0$ zu dem Niveau v' der drei Elektronenzustände des angeregten Radikals entstehen. Die Banden der ersten Serie sind diffus; sie bezeugen eine Prädissoziation des Moleküls nach Schema 1:

$$C_6H_5NH_2 + h\nu \rightarrow C_6H_5NH + H. \qquad (I)$$

Bei $\lambda = 1900$ Å beginnt die eigentliche kontinuierliche Absorption, was wahrscheinlich dem direkten Zerfall des Anilins nach Schema I zugeschrieben werden muss. Im Spektrum des Magnesiumfunkenlichtes, das durch einen Gelatinefilm filtriert wird, fehlt das Gebiet mit kürzeren Wellenlängen, als $\lambda = 2450$ Å. Es ist daher schwer zu erwarten, dass bei Zimmertemperatur ohne autokatalytischen Einfluss ein Zerfall des Moleküls nach Schema I stattfinden wird. Bei hohen Temperaturen kann ein derartiger Zerfall auch an einem isolierten Molekül stattfinden*.

Bei Belichtung des Anilins im Gebiet der diffusen und kontinuierlichen Absorption der Phenylgruppe, zerfällt Anilin nach Schema II und III:

$$C_6H_5NH_2 + h\nu \rightarrow C_6H_4NH_2 + H \ (II) \qquad C_6H_5NH_2 + h\nu \rightarrow C_6H_5 + NH_2. \ (III)$$

Der beim photochemischen Zerfall des Anilins freiwerdende atomare Wasserstoff wird entweder zu molekularem Wasserstoff recombiniert, oder bildet mit der Aminogruppe Ammoniak. Zur Bildung von Azophenin müssen die Radikale—$C_6H_4NH_2$, C_6H_5NH—und—C_6H_4NH—führen. Ausser diesen Verbindungen können im Reaktionsgefäss Benzol, Diphenyl, Hydrazin u. a. gebildet werden. Die Anwesenheit dieser Stoffe konnte jedoch spektroskopisch nicht festgestellt werden. Die Untersuchung des Benzols ([19], [20]) ergab, dass bei der Belichtung von Benzoldämpfen im Gebiet der diffusen und kontinuierlichen Absorption ein photochemischer Zerfall nach dem Schema $C_6H_5 + h\nu \rightarrow C_6H_5 + H$ stattfindet.

* Es wurde gezeigt ([4]), dass chemisch reines Anilin, das sich in einem gut evakuierten Quarzgefäss befindet, beim Erwärmen allein, ohne Belichtung, während 10 Stunden auf 400° C keine chemische Umwandlungen erleidet.

Bei Temperaturerhöhung wird die Grenze der Prädissoziation und der kontinuierlichen Absorption des Anilins in Richtung des langwelligen Gebiets verschoben. In Abb. 2 sind die Absorptionsspektra der Anilindämpfe (p=0.3 mm) bei 18, 150, 250 und 400° C wiedergegeben. Bei 18° beginnt die kontinuierliche Absorption bei 2632 Å, bei 400° aber wird nur ein kontinuierliches Band beobachtet. Eine analoge Verwischung des Absorptionsspektrums hat Henri ([21]) an Benzol beobachtet.

Bei hohen Temperaturen kann das Molekül unter der Wirkung derjenigen Wellenlängen dissoziieren, die bei niedrigen Temperaturen das Molekül nur anregen konnten*. Diese Temperaturabhängigkeit des Absorptionsspektrums ist die Ursache der Temperaturabhängigkeit der photochemischen Reaktion und photothermischen Auslöschung der Fluoreszenz des Anilins ([4]).

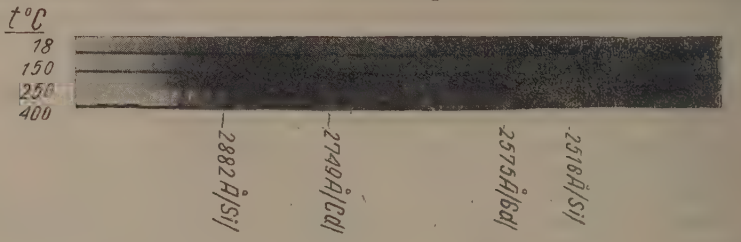


Abb. 2.

Der Zerfall des Anilins bei Zimmertemperatur erfolgt wahrscheinlich infolge der zeitweisen Anreicherung der Wärmeenergie an der zu zerreissenden Bindung des angeregten Moleküls, da die kürzesten Wellen des Magnesiumfunkenlichtes, das durch Gelatinefilme filtriert wird, nicht in das Gebiet der diffusen bzw. kontinuierlichen Absorption reichen. Der Energiedefekt beträgt etwa 2 kal/Mol., d. h. eine Grösse, die leicht durch das Molekül mit vielen inneren Freiheitsgraden gedeckt wird.

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Übersetzt von K. Packendorff.

* Interessant wäre es den photochemischen Zerfall von erhitzten Benzoldämpfen bei Belichtung mit Kadmiumfunken (2300—2150 Å), die bei Zimmertemperatur wirkungslos sind ([19]), zu verfolgen.

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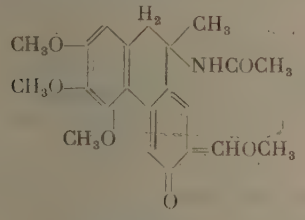
BIOCHEMISTRY

METHOXYL DERIVATIVES OF BENZENE AND NAPHTHALENE STUDIED WITH REGARD TO THEIR POLYPLOIDOGENIC ACTION ON PLANTS

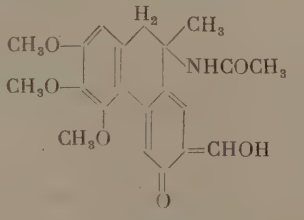
By A. SHMUCK and A. GUSSEVA

(Communicated by A. A. Richter, Member of the Academy, 21. XII. 1940)

In studying the polyploidogenic action on plants of various naphthalene derivatives with reference to their chemical structure, we have disclosed that the methoxyl and ethoxyl derivatives of α-naphthol are endowed with considerable activity, and so too are certain esters of α-naphthoic acid (1). Colchicine, which is notoriously active in this respect (2), has also three methoxyl groups in its molecule. But saponification of only one of them results in formation of the entirely inactive colchiceine (3) out of colchicine.



colchicine



colchiceine

So we have come to the assumption that the polyploidogenic activity of an aromatic compound depends strongly upon the presence and position of methoxyl groups within the molecule.

Accordingly we have prepared and put to the test a series of methoxyl derivatives of benzene and naphthalene varying in respect to the number and position of their methoxyl groups. It has been found in that way that, like the phenols themselves, their methyl esters are deprived of any polyploidogenic activity.

The test was run with wheat seedlings, the compounds tried being introduced as ether solutions on filter paper placed into Petri dishes. After evaporation of the ether, the paper was wetted with water and the seeds were laid out on it. A uniform dosage was applied in all cases: 1, 5 and 10 mg of the substance concerned to a dish with 10 seeds. The polyploidogenic effect was judged upon by morphological modifications and from the data of cytological analysis (carried out by M. Maslova) which had made the basis for final decisions (Table 1).

Table 1

Inactive	Inactive
Phenol	1,4 Dimethoxybenzene
Anisol	1,2,3 Trioxybenzene (pyrogallol)
1,2 Dioxybenzene (pyrocatechine)	1,2,3 Trimethoxybenzene
1,2 Dimethoxybenzene	1,3,5 Trioxybenzene (fluoroglucine)
1,3 Dioxybenzene (resorcine)	1,3,5 Trimethoxybenzene
1,3 Dimethoxybenzene	1,2,4 Trioxybenzene (oxyhydroquinone)
1,4 Dioxybenzene (hydroquinone)	1,2,4 Trimethoxybenzene

It will be obvious from the results that no polyploidogenic power could be imparted to these derivatives by one, two or three methoxyl groups being introduced in any possible position into the molecule of benzene.

Many methoxyl derivatives of naphthalene were prepared and tried by us, but most of them proved inactive either. Those which were active, however, showed a definite arrangement of methoxyl groups (Table 2).

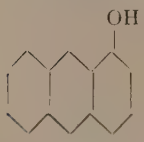
Table 2

1 Oxynaphthalene (α-naphthol)	inactive	2,3 Dimethoxynaphthalene	inactive
1 Methoxynaphthalene .	very active (polyploids)	2,7 Dioxynaphthalene . .	»
2 Oxynaphthalene (β-naphthol)	inactive	2,7 Dimethoxynaphthalene	»
2 Methoxynaphthalene .	»	1,8 Dioxynaphthalene . .	»
1,4 Dioxynaphthalene .	»	1,8 Dimethoxynaphthalene	active (multinuclear cells, polyploidy)
1,4 Dimethoxynaphthalene	active (disturbed cell division; multinuclear cells; no polyploidy)	2,6 Dioxynaphthalene . .	inactive
		2,6 Dimethoxynaphthalene	»
2,3 Dioxynaphthalene .	inactive		

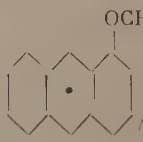
The conditions under which the methoxyl derivatives of naphthalene were tested were the same as in the case of benzene derivatives. Unfortunately, we were not able to establish the activity of trimethoxynaphthalenes which are very easily oxidizable in the air. All preparations of dioxynaphthalene, like those of oxybenzene, were inactive.

Comparing the polyploidogenic activity and chemical structure of different methoxynaphthalenes, we found complete correspondence with the regularity established previously: active are the derivatives in α-position, while those in β-position are inactive([4]). In point of biological activity α-derivatives of naphthalene appear therefore to be very different from its β-derivatives, which falls in line with considerable physical and chemical differences between α and β-positions reported in the molecule of naphthalene.

In testing α-anthrol and its methyl ester prepared by us, we found these two derivatives of anthracene to have produced no polyploidogenic action whatever:



α-anthrol

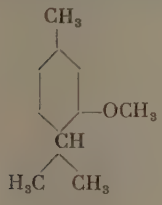


α-methoxyanthracene

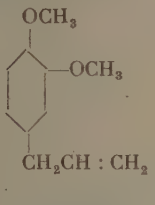
It will be apparent that when introduced into the molecule of naphthalene, the methoxyl groups do not necessarily bring with them the polyploidogenic power. Their position in the molecule is here a factor of prime importance. The most active is the methyl ester of α-naphthol, while 1.4 and 1.8 dimethoxynaphthalenes are endowed with far lower activity. In β-position all derivatives of naphthalene are inactive.

Gavaudan([5]) has reported polyploidogenic activity in apiole, a substance contained in the essential oil from parsley. Apiole is a derivative of tetraoxy-

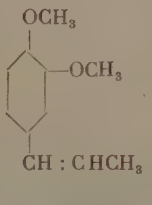
benzene and is high in methoxyl groups. Lefevre([6]) has discovered similar biological powers in anetole (1-methoxy-4-propenylbenzene). These data induced us to try some preparations which were methoxyl derivatives of benzene with different side chains. The substances put to the test were: thymol, guiacol, eugenol, isoeugenol, khavicol and methyl esters of thymol, eugenol, isoeugenol, carvacrol and khavicol.



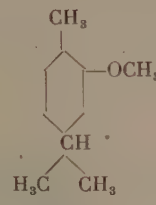
methyl ester of thymol



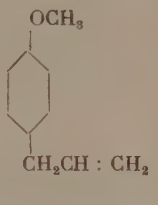
methyl ester of eugenol



methyl ester of isoeugenol



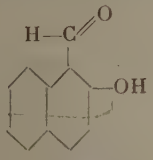
methyl ester of carvacrol



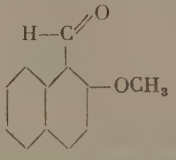
methyl ester of khavicol

Most of them proved very toxic to the seedlings of wheat, but if applied in low proportions they failed to produce any polyploidogenic effect. Remarkable was the action of methyl carvacrol and methyl khavicol, which caused no appreciable deviations in cell division, but at the same time induced strong morphological modifications in the development of leaves, reducing their length and increasing their breadth considerably.

In the molecule of colchicine a very mobile carbonyl group with the properties of aldehyde nature appears to be present along with the methoxyl groups. It was therefore interesting to test naphthaldehyde and its derivatives for polyploidogenic activity. For this purpose we obtained 2-oxy-1-naphthaldehyde by subjecting β-naphthol to the action of an alkaline solution of chloroform([7]).



2-oxy-1-naphthaldehyde



2-methoxy-1-naphthaldehyde

This compound had no polyploidogenic activity whatsoever in relation to young seedlings of wheat. Neither had the 2-methoxy-1-naphthaldehyde obtained from it produced the characteristic polyploidogenic effect on wheat seeds during their germination.

The conclusion to be drawn from all these tests is that it is only in a small number of cases that the methoxyl groups introduced into the molecule of benzene or naphthalene will impart the polyploidogenic activity to the derivatives obtained in this way; only the α-derivatives of naphthalene have been found to be active.

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BIOCHEMISTRY

ACTIVITY OF POLYPLOIDOGENIC COMPOUNDS AS INFLUENCED BY HYDROGENATION

By A. SHMUCK and A. GUSSEVA

(Communicated by A. A. Richter, Member of the Academy, 21. XII. 1940)

Many chemical compounds of aromatic structure able to influence animal and vegetable cells and tissues in a specific way lose this ability, partly or even entirely, in response to hydrogenation. In the case of carcinogenic compounds such effect of hydrogenation has been reported by many investigators(1). Koepfli, Thimann and Went(2) have found that the activity of indolacetic acid as a root-formation hormone is completely removed by hydrogenation. Inactivity has also been reported by Kögl(3) in dihydroauxine.

To promote root formation is also to a great extent a quality of the naphthylacetic acid and here, too, the effect of hydrogenation, as our observations have shown, is to detract much from this quality.

In view of this evidence we set ourselves the task to investigate the effect of hydrogenation on the biological activity of some of the most active polyploidogenic compounds. As colchicine was known to hold a place of prominence among them, we started our tests with this substance.

Hydrogenation was effected by hydrogen with platinum-black in a solution of icy acetic acid, according to Windaus (4). What we had obtained in this way was octohydrocolchicine. The preparation was tried on wheat seeds during their germination and was lacking any polyploidogenic power.

Octohydrocolchicine's action on animal tissues had been tested before by Brues and Cohen(5) who had stated a very considerable reduction in its power to influence the cell division (Table 1).

Table 1

	Minimum dose, mgs in 100 g	Optimum dose, mgs in 100 g	Lethal dose, mgs in 100 g
Colchicine	0.02	0.1	0.5
Octohydrocolchicine	3.0	10.0	—

Thus, the effect of hydrogenation on colchicine is greatly to reduce its activity in relation to animal cells and makes it entirely inactive in relation to vegetable cells.

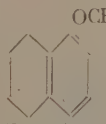
Similar results have been obtained by hydrogenation of acenaphthene and α-methylnaphthalene, which are also powerful inducers of polyploidy in plants. Hydrogenation of these hydrocarbons had been carried on, according to Bamberger(6); the tetrahydroacenaphthene and methyltetrahydronaphthalene so obtained were both completely inactive.

Lack of polyploidogenic power has been reported by us previously([7]) also in tetrahydronaphthalene and decahydronaphthalene; this in face of the fact that the naphthalene itself is endowed, albeit to a slight extent, with such power.

To sum up, all active hydrocarbons able to induce polyploid modifications in plants during cell division are deprived of this quality by hydrogenation of their molecules.

In our previous experiments we had found a considerable polyploidogenic activity in α-bromonaphthalene([8]) and in the methyl ester of α-naphthol([9]). It was interesting therefore to put to the test the activity of their hydroderivatives.

Ar-α-bromotetrahydronaphthalene



Ar-α-methoxytetrahydronaphthalene

The synthesis of Ar-α-bromotetrahydronaphthalene was brought about, according to Smith ([10]), by way of obtaining α-aminotetrahydronaphthalene from α-naphthylamine by reduction and carrying on the reaction further, according to Sandmeier, through diasocompounds to α-bromotetrahydronaphthalene. As a by-product of this reaction α-oxytetrahydronaphthalene was obtained and used for the preparation of the respective methyl ester.

Table 2

	Minimum active dose, mgs in a Petry dish	Lethal dose, mgs in a Petri dish
α-methoxynaphthalene	0.2	2.0
Ar-α-methoxytetrahydronaphthalene .	2.0	10.0
α-bromonaphthalene	< 0.1	5.0
Ar-α-bromotetrahydronaphthalene .	2.0	5.0

Both of these functional derivatives of tetrahydronaphthalene when tested on wheat seedlings were found to have produced a polyploidogenic effect, but by far not so strong one as that produced by α-bromonaphthalene and α-methoxynaphthalene on similar seedlings (Table 2).

It will be obvious that while the hydrocarbons endowed with polyploidogenic power as surely lose it in response to hydrogenation as does colchicine, the derivatives of hydrogenated hydrocarbons which have preserved their functional groups in the aromatic nucleus retain some part, if not a very considerable one, of the former activity.

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BIOCHEMISTRY

CHEMISTRY AND MECHANICS OF THE MUSCLE STUDIED ON MYOSINE THREADS

By W. A. ENGELHARDT, M. N. LJUBIMOVA and R. A. MEITINA

(Communicated by A. N. Bach, Member of the Academy, 31. XII. 1940)

Myosine is regarded as the contractile substance of the muscle fibre, and the elementary muscular act consisting in contraction and relaxation is ascribed to changes in the molecular or micellar structure of myosine, resulting from the interaction of myosine with substances formed or disappearing in the metabolic processes associated with muscular activity (we shall refer to them as «activity substances»). This view is generally accepted, but has never been proved experimentally. Attempts to demonstrate the influence of chemical factors involved in muscular activity on the physical properties of myosine were restricted to the study of the solubility of myosine([1]) and gave no positive results. In the experiments reported here an attempt was made to investigate the influence of possible «activity substances» on the mechanical properties of the contractile substance of muscle.

Weber([2]) has shown that, when injected in distilled water, myosine solutions form threads of considerable strength. Such threads give a characteristic X-ray diagram closely resembling the diagram given by muscle tissue. In this respect the myosine thread may be regarded as a very adequate model, reproducing the molecular structure responsible for the changes of state of the contractile substance of muscle tissue. Previous investigations([3]) have led to the conclusion that myosine possesses a specific enzymic activity, namely, that of adenosinetriphosphatase. This activity, as we have found, can be detected also in the myosine threads.

We undertook experiments in order to investigate, whether or not the myosine thread, possessing the fine structure of the contractile substance of muscle and the original katalytic activity of myosine, may be regarded as a mechano-chemical model of the muscle fibre, capable to reproduce its fundamental feature—the alteration of mechanical properties in response to chemical influences. The result obtained was a positive one.

For the study of the mechanical properties of myosine threads the arrangement shown in Fig. 1 was used.

The thread is fastened by means of a clamp to the lever of a torsion balance and can be immersed in the solution of the substance under investigation. By moving the balance handle varying tension is applied to the thread, and the displacement of the lever caused by the distention of the thread is recorded on a millimeter scale, upon which a reflection is projected from a mirror fastened to the lever. Either the extension under a given load or the load

required to produce a given extension can be measured; in most cases the first way was adopted.

Potassium, calcium, phosphate ions, creatine, carnosine, adenylic acid (most of them taken in concentrations comparable to those found in the muscle) exerted no significant influence on the mechanical properties of the myosine thread. Acetylcholine was also ineffective. Contrary to expectation, changes of hydrogen ion concentration within a fairly wide range from pH 5.0 to 7.5 have but a very slight effect; beyond these limits the thread begins to disintegrate and breaks. So far, only one substance has been found to alter profoundly in physiological concentrations the properties of a myosine thread. This substance is adenosinetriphosphoric acid.

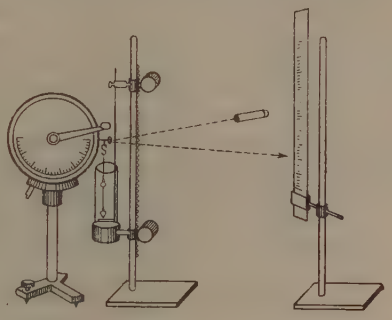

Fig. 1.

When a myosine thread is immersed in a solution of adenosinetriphosphate (in most experiments a solution of approximately 0.0025 M was used), the extensibility of the thread is increased by some 50 to 100 and more per cent as compared with the extension under the same load in water or dilute salt solution. When the thread is returned to water or dilute saline, the extensibility tends to return to the initial value, though not always attaining it. The results of a typical experiment are shown in Fig. 2.

When myosine threads are kept for a longer time under water, they gradually lose the capability to alter their mechanical properties under the influence of adenosinetriphosphate. As a rule, this loss goes along with the loss

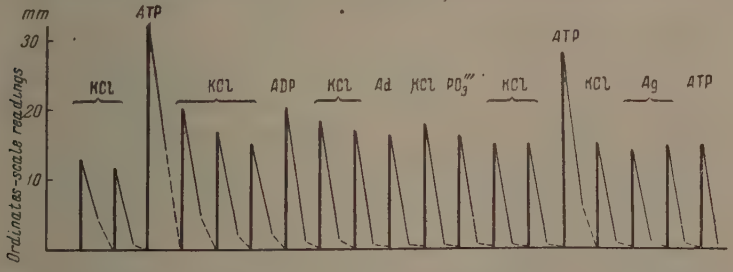

Fig. 2. Tension on thread in all cases 200 mg. ATP—adenosinetriphosphate; ADP—adenosinediphosphate; AD—adenylic acid; PO_3—solution of phosphates; Ag—$AgNO_3$. Ordinates—scale readings, mm.

of the enzymatic (adenosinetriphosphatase) activity. In experiments on myosine solutions it has been found that silver ions in very low concentrations inhibit completely the adenosinetriphosphatase activity of myosine. When treated with $AgNO_3$ myosine threads cease to show increased extensibility in adenosinetriphosphate solutions, whereas their extensibility in water or saline remains practically unchanged (Fig. 2, right-hand section of diagram).

One has the impression that the mechanical reactivity of the myosine thread towards the influence of adenosinetriphosphate is present as long as the enzymatic activity exists. It is premature, though very attractive, to conjecture on a causal relation between the two properties, further experimental evidence being required for a definite conclusion. At present it is safer to assume that the mechano-chemical reactivity of the thread is associa-

ted with the native state of myosine, and the enzymatic activity is also to be regarded as an indication of the non-denatured state of the protein.

The circumstance that adenosinetriphosphate produces not a contraction, but a relaxation of the myosine thread, does not impair the possible physiological significance of the facts here described. According to modern views, it is just with the recovery processes that the chemical events in muscle are mainly connected, not with the contraction.

The experiments here reported have permitted for the first time to demonstrate the influence of chemical agents on the primary contractile substance of the muscle. It seems probable that the myosine thread will be a convenient object for the study of what may be defined as the mechano-chemistry of the muscle, the interrelations between chemical events and mechanical properties. It will be of interest to investigate the action of enzymatic systems, of contraction-producing poisons, of surface-active substances etc. Experiments along these lines are in progress.

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MINERALOGY

ON THE QUESTION OF THE RELATIONS OF CLINOENSTATITE TO ENSTATITE AND DIOPSIDE

By D. S. BELIANKIN, Corresponding Member of the Academy, and B. V. IVANOV

It would seem that this question should be regarded as definitely settled long since, thanks to the well-known classical work by Bowen which gave an illustrative and clear evidence of the complete isomorphism of clinoenstatite and diopside and, consequently, of its dimorphism with enstatite. Yet, we come back to the question in connexion with a study we have just achieved of a very curious mineral formation, produced in an open-hearth furnace of the Stalin works at the contact of the Dinas brick and magnesite in the brickwork of the latter. Owing to the high temperature of the furnace in course of melting of the metal, both above-mentioned refractories interacted, and as a product of this interaction a plate was formed, several centimeters thick, consisting of large skeleton crystals of clinoenstatite, the interstices between the crystal branches of which were filled with a more finely-crystalline mixture of augite, cristoballite and magnetite. It appeared that these clinoenstatite branches are not simply monomineral, but that their surface is overgrown with shells of enstatite-diopside which are in parallel crystallographic position to them and pass in turn, further on, at the finishing marginal part of the branch into a yellow-green augite, of about the same character and composition as the augite of the interskeletal mass of the crystal. The most typical feature here is that while augite is connected with enstatite-diopside at least partly by continuous and gradual transitions, the transitions from enstatite-diopside to clinoenstatite are always abrupt, proceeding by leaps and bounds, as in two different quite independent minerals. Clinoenstatite shows its usual, extremely thin polysynthetic twinning striation along 100; enstatite-diopside also shows twinning along this plane, but it is either simple or consisting of a few relatively broad double twinning lamellae (see Bowen, 1914, p. 246).

In the table are given for the sake of comparison the optical constants of the three pyroxene mineral formations described above.

The above-described independent shelly crystallization of enstatite-diopside around clinoenstatite closely resembles a similar shelly crystallization of potash-anorthoclase (orthoclase) or microcline around plagioclase crystals (oligoclase or andesite), frequently encountered in nature. And just as this fact does not prevent in other cases, under different physical-chemical conditions, the joint crystallization of $KAlSi_3O_8$, $NaAlSi_3O_8$ and $CaAl_2Si_2O_8$ molecules in the form of fully homogeneous monomineral soda-orthoclases or anorthoclases, in the case of our pyroxenes under the influence of the very same

Optical Constants of Pyroxenes in the Mineral Formation of the Open-Hearth Furnace of the Stalin Works

Minerals	Plane of optical axes	CNg	$2V$	Ng	Np
Clinoenstatite	$\perp$ 010	21.5°	44°	1.662	1.651
Enstatite-diopside	—	35—44°*	0°	1.686	1.658
Augite	$\parallel$ 010	49—52°*	68—82°*	1.742	1.702

* The values increase in the direction from the interior parts to the exterior parts of crystal formation.

physical-chemical conditions may be formed either shelly forms of enstatite-diopside around enstatite or clinoenstatite, and Bowen's completely homogeneous monomineral enstatite-diopside mixtures. The difference is only in the sequence of crystallization: from plagioclase to anorthoclase and microcline in feldspars and from enstatite or clinoenstatite to enstatite-diopside in pyroxenes.

As to the physical-chemical conditions responsible for such a different crystallization of calcareous-magnesial pyroxenes in our case and in that of Bowen, it is natural to suggest in the first place the influence of temperature, which in our case is lower owing to the presence of alumina and iron oxides in the crystallizing calcareous-magnesial-silicate mass, as shown by the following bulk analysis of our mineral formation made by V. M. Nekrasova:

SiO_2	58.57%	TiO_2	0.90%	Al_2O_3	1.34%
Fe_2O_3	5.19%	FeO	2.75%	MnO	0.63%
MgO	27.12%	CaO	3.30%	Na_2O	0.19%
K_2O	0.10%	P_2O_5	0.11%	Total	100.20%

The calculation of this analysis for minerals, in full agreement with the direct quantitative determinations under the microscope, yielded the following figures: clinoenstatite 60%, enstatite-diopside+augite 20%, cristobalite+magnetite 20%.

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GENETIK

ÜBER DIE RELATIVE GRÖSSE DER WURZEL BEI IM ERSTEN LEBENSJAHRE BLÜHENDEN EXEMPLAREN VON KOK-SAGHYZ

Von A. J. KUPZOW

(Vorgelegt von A. A. Richter, Mitglied der Akademie, den 25. XII. 1940)

Bei der Selektion und Samenzüchtung von Kok-saghyz ist die Frage über den Zusammenhang zwischen der Grösse der Wurzel und dem Blühen im ersten Lebensjahre von wesentlicher Bedeutung. Bis jetzt ging man bei der Selektion und Samenzüchtung dieser Pflanze ganz richtig davon aus, dass die grössten Wurzeln der Gruppe angehören, die im ersten Lebensjahre nicht blüht. Dementsprechend wird bei der Samenzüchtung von Kok-saghyz gegenwärtig die Sammlung der Samen von den Plantagen des zweiten Lebensjahres dieser Kultur vorgenommen; bei der Selektion werden die Schemata der Auswahl benutzt, die man gewöhnlich bei den Wurzelfrüchten anwendet, d. h. es werden die ausgewählten Wurzeln zur Sammlung der Samen im zweiten Lebensjahre verpflanzt, ohne die Sammlung der Samen im ersten Jahre vorzunehmen. Dies ist auf Grund einer Reihe von Versuchen ([2],[3],[4]) in der Literatur beleuchtet; ausführlicheres Material über diese Versuche findet man in den Berichten (Manuskript) des Institutes für Kautschukpflanzen. Verschiedene Autoren ([5]) jedoch, die mit der Literatur dieser Frage nicht genügend bekannt sind und sich nicht in das Wesentliche der Frage vertiefen, treten zu empirisch an ihre Aufklärung heran und rufen dadurch den Eindruck eines Widerspruches zwischen ihren Ergebnissen und den Annahmen anderer Autoren hervor. Das zwingt uns hier kurz diejenigen Gesetzmässigkeiten darzulegen, welche bei Kok-saghyz mit dem Blühen im ersten Lebensjahre verbunden sind.

Gewöhnlich kann man auf den Plantagen des ersten Lebensjahres von Kok-saghyz das Blühen eines Teiles der Pflanzen beobachten. Die Prozentzahl der im ersten Lebensjahre blühenden Kok-saghyzpflanzen ist schwankend; sie erhöht sich bei günstigen Bedingungen der Jarovisierung und bei Anbau auf hoch fruchtbaren Böden, ähnlich wie dies etwas später bei der Zuckerrübe([1]) beobachtet wurde. Da die Fruchtbarkeit des Bodens, die das Blühen beschleunigt, gleichzeitig auf die Grösse der Wurzel günstig einwirkt, sind die Plantagen, die im ersten Lebensjahre reichlicher blühen, gewöhnlich durch grössere Wurzeln (Tab. 1) charakterisiert; die zurückgebliebenen kleinen Pflanzen gelangen in der Regel im ersten Lebensjahre nicht zur Blüte. Dadurch wird das phaenotypische Verhalten dieser Erscheinung charakterisiert.

Jedoch sogar auf sehr fruchtbaren Böden bei äusserst günstigen Bedingungen finden sich nicht selten Exemplare von Kok-saghyz, die obgleich weit nicht unterentwickelt sind, im ersten Lebensjahre nicht blühen. Auf diese Individuen ist die Selektion und Samenzüchtung dieser Kultur eingestellt.

Tabelle 1

Prozentsatz der im ersten Lebensjahre blühenden Kok-saghyzpflanzen und Grösse der Wurzel unter verschiedenen Bedingungen

	Ustimowka (bei Poltawa) 1935	Moskau 1934	Tomsk 1939
Prozentsatz der im ersten Lebensjahre blühenden Exemplare . .	16.1	34.2	50.8
Diameter des oberen Teiles der Wurzel in mm	7.3	10.2	10.8

Diese Exemplare sind durch einen längeren Entwicklungszyklus charakterisiert und zeichnen sich durch eine gefiederte Blattspreite der oberen Blätter der Rosette aus. Die ersten Blätter von Kok-saghyz sind, wie bekannt, immer ganzrandig und manche Pflanzen behalten diese Form in der Ontogenese ihrer oberen Blätter, bei anderen sich später entwickelnden Individuen sind die Blätter gelappt, und bei einem anderen Teil schliesslich gehen die aufeinanderfolgenden Blätter, noch bevor die Pflanzen zur Blüte gelangen, von den ganzrandigen zu den gelappten und dann zu den fiederförmig-geteilten über. Die Beobachtungen haben gezeigt, dass die Kok-saghyz Exemplare mit einem längeren ontogenetischen Entwicklungszyklus im ersten Lebensjahre gewöhnlich nicht blühen, eine grössere Anzahl von Blättern in der Rosette und gleichzeitig eine dickere Wurzel besitzen. Augenscheinlich sind bei Kok-saghyz zwei Gruppen von Genotypen vorhanden, ebenso wie bei *Daucus carota* L., *Beta maritima* L., *Cichorium intubus* L. und bei einer Reihe anderer zwei- und mehrjähriger botanischer Arten: eine, die leichter und schneller zur generativen Phase übergeht und eine bedeutende Menge ihrer Assimilate dazu verbraucht; die andere, die weniger zur frühen Fruchtbildung geneigt ist, begnügt sich gewöhnlich im ersten Lebensjahre mit der Bildung bestimmter Vorräte organischer Stoffe in der Wurzel. Darin besteht das genetische Verhalten dieser Erscheinung, die man an dem Material der Selektions-Pflanzschulen bei relativ regelmässiger seltener Verteilung der Pflanzen am deutlichsten erkennen kann (Tab. 2). So entstand die von den Selektionären üblich angewandte Formel von der stärkeren Dicke der Wurzel bei im ersten Lebensjahre nichtblühenden Exemplaren von Kok-saghyz, die das genetische Verhalten dieser Erscheinung ausdrückt, vorausgesetzt, dass die Entwicklungsbedingungen bei allen zu vergleichenden Individuen die gleichen und optimalen sind.

Tabelle 2

Dicke der Wurzel (Durchmesser des oberen Teiles in mm) bei verschiedenen Kok-saghyz Formen im ersten Lebensjahre

	Individuen mit oberen Blättern					
	Ganzrandigen		Gelappten		Gefiedert-geteilten	
	blühende	nichtblühende	blühende	nichtblühende	blühende	nichtblühende
Dschun (bei Taschkent):						
1936	6.1	6.6	7.4	7.9	9.2	9.4
1937	8.4	6.2	8.8	8.6	10.0	10.7
Michnevo (bei Moskau):						
1936	12.9	13.5	12.9	13.3	13.4	14.0
1939	15.4	15.5	15.8	16.5	15.8	16.4
Tomsk, 1939	10.7	10.3	11.1	11.2	11.4	12.8

Die konkrete Wirklichkeit ist komplizierter als die abgeleiteten Abstraktionen. Die Gruppe der im ersten Lebensjahre blühenden Exemplare von Kok-saghyz ist relativ einheitlicher nach ihrem Bestand, da sie hauptsächlich normal entwickelte Pflanzen von Genotypen mit relativ kurzem Entwicklungszyklus enthält. Was die Gruppe der im ersten Lebensjahre nicht blühenden Exemplare anbetrifft, so enthält sie, einerseits, normal entwickelte Pflanzen mit einem langen Zyklus der Ontogenese und, andererseits, alle zurückgebliebenen Individuen, die sich unter ungünstigen Bedingungen befanden. Deshalb wurde beobachtet, dass die im ersten Lebensjahre blühenden Exemplare von Kok-saghyz oft nach ihrer Wurzeldicke eine Mittelstellung einnehmen, während die kleineren und dickeren Wurzeln in diesem Alter nicht blühen(2).

Dies erklärt, dass die Mittelgrösse der Wurzeldicke bei im ersten Lebensjahre nichtblühenden Exemplaren von Kok-saghyz im Verhältnis zu der Wurzeldicke bei blühenden Pflanzen sowohl kleiner sein kann (wenn die Plantage dicht, nicht gleichartig und in schlechtem Zustande ist mit einer grossen Menge von deprimierten sich anormal entwickelnden Pflanzen), als auch grösser (wenn die Plantage vereinzelt, gleichartig ist und sich unter günstigen Bedingungen mit einer unbedeutenden Anzahl von kleinen zurückgebliebenen Pflanzen befindet) (Tab. 3).

Tabelle 3

Relative mittlere Wurzeldicke (Durchmesser des oberen Wurzelteiles in mm) bei im ersten Lebensjahre blühenden und nichtblühenden Individuen von Kok-saghyz bei verschiedenen Bedingungen in Tomsk

	Exemplare	
	Blühende	Nichtblühende
1939 (vereinzelte Plantage bei günstigen Bedingungen)	11.2	11.7
1940 (dichte Plantage bei ungünstigen Bedingungen)	7.9	6.7

Solche Widersprüche sind vollkommen gesetzmässig entsprechend den verschiedenen Bedingungen der Saat und des Wetters verschiedener Gebiete in einzelnen Jahren; sie sind durch die Ergebnisse der vorhergehenden Arbeiten leicht zu erklären und erschüttern in keiner Weise die Annahme, dass bei normaler und guter Entwicklung die grösseren Exemplare von Kok-saghyz der im ersten Lebensjahre nichtblühenden Gruppe angehören.

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GENETICS

SIGNIFICANCE OF HAEMATOLOGICAL STUDIES IN HORSE BREEDING*

By H. F. KUSHNER

(Communicated by I. I. Schmalhausen, Member of the Academy, 13. XII. 1940)

It has been proved by many physiologists and biochemists ([1–5]) that metabolism in the muscular cell, and consequently the productivity of this cell, is highly dependent on the oxidizing properties of the blood.

Our own haematological studies of British thoroughbreds at the Moscow Hippodrome in the summer of 1938 proved the existence of a high rate of correlation between the oxidizing properties of the blood and the racing qualities of the horses. Thus, the rate of correlation between haemoglobin value and speed in two groups of racing horses was as follows: for the first group of 38 mares two years of age 0.414 ± 0.13; for the second group 24 stallions three years of age 0.666 ± 0.14. We do not know of any other exterior or physiological factor in homogeneous groups of animals that shows such a high rate of correlation with speed as the red blood factors examined by us. Indeed, the best track performers at the hippodrome, as a rule, showed high haemoglobin value. Thus, while the average haemoglobin value for 4-year-old stallions was 105.28% (by Sahli), the best racing stallions—Sector II and Sabo—showed, respectively, 112 and 112.5%. The same picture was to be observed in the 3-year-old stallions. Whereas the average haemoglobin value for this group was 100.8%, Komar, Espadron and Brighton Boy, the best stallions in this group, showed 109. 111, 113%, respectively. Finally, in the 3-year-old mares group, with an average haemoglobin value of 94.46%, Nasha Gordost and Phenella, the best winners, showed 104 and 108%, respectively.

According to the rules prevailing at the Moscow Hippodrome, young horses with the best records of the season are retained for another year and sometimes even for two more years. The selection is made by specialists who guide themselves solely by the results of the races. Some of the younger horses (two and three years of age) examined by us in 1938 were retained at the hippodrome. And thus, in 1939 and 1940 we had the opportunity of observing the correlation between the blood value of the horses at an early age and their subsequent racing qualities. This provided an answer to the question as to the value of haematological study of animals at an early age as an additional criterion (over and above the usual zootechnical methods) permitting of

* In our studies we were greatly assisted by G. G. Khitenkov and K. N. Schnell, specialists of the Institute for Horse Breeding and of the Moscow Hippodrome. O. N. Kitaieva and Ch. B. Alperovich helped in taking the analyses.

a prognostication of the future performance of the horse, which is naturally of great importance to the breeder.

Table 1 shows the blood value of the horses examined by us at the age of two and three years which were selected for the next year's races by the specialists of the hippodrome independently of the results of our analyses. For the sake of comparison the blood value of the rejected horses is also given.

Table 1

Blood Value of Two Groups of Horses Examined in 1938

	Number	Haemoglobin in % (by Sahli)	Number of erythrocytes (in millions)	Alkalinity (in mg%)
I. Mares, 2 years of age: .				
1) Selected for next year's races	12	96.18	11.89	674
2) Rejected	36	92.9	11.45	660
II. Stallions, 3 years of age:*				
1) Selected for next year's races	6	102.2	12.2	728
2) Rejected	24	98.5	11.54	676

* Four horses of this group (Komar, Brighton Boy, Espadron and Bochar), excellent racers, were not selected, however, for next year's races for the following reasons: temporary lameness and injury to hoof; they were also needed for breeding purposes.

From Table 1 it may be seen that in both groups the horses selected for their racing records in the past year showed the highest blood value. This is as it should be, for the performance of an animal depends on the intensity of the metabolic activity in the muscular cell, which in its turn depends on the oxidizing properties of the blood.

The correlation between the blood value of animals at an early age and their subsequent performance is brought out more clearly when we examine superior individuals. In 1938, when we were examining a group of two-year-old mares with an average haemoglobin value of 93.8% and an erythrocyte content of 11.56 million, our attention was attracted by Zygota, a mare which showed a haemoglobin value of 102.5% and an erythrocyte content of 12.39 million. Zygota turned out a remarkable racer; in 1939 she won the Grand Prize in a race of 3-year-old mares, and the All-Union Derby. Two other mares in this group, Mara and Tropa, which in 1938 showed a haemoglobin value of 100%, similarly gave a good account of themselves at the 1939 races.

Of the stallions examined by us at the age of two and retained by the hippodrome for next year's races, those with the highest blood value carried off the highest prizes. Table 2 shows the blood value of these horses. This group had an average haemoglobin value of 93.7%, an erythrocyte content of 11.5 million and an alkalinity of 673 mg%.

On the ground of all these data, and some other besides, we come to the conclusion that of the large number of examined horses those which originally showed higher blood value fared best in most of the cases. There were naturally some exceptions, as e. g. Express, a stallion whose blood value at the age of two was below the average for its group, which, nevertheless, established a new All-Union record for speed, having covered 2400 m in 2 min. 31′ sec. In such cases (provided the low figures in our analysis were not due to some cause which incidently was overlooked by us) it is to be assu-

Table 2

Blood Value of Best Stallions at 2 Years of Age and Their Subsequent Racing Record

Name of horse	Age at which it won prize (years)	Distance covered (in m)	Time	Blood value		
				Haemoglobin % (by Sahli)	Number of erythrocytes (in millions)	Alkalinity (in mg %)
Horizon	4	2000	2.04[5]	108	13.59	760
Globus*	3	2400	2.32[5]	101	10.39	740
Hamid-Bi	3	2400	2.32[4]	97.5	12.74	740
» »	4	4800	6.03[5]	97.5	12.74	740
Huguenot	4	2400	2.32[7]	97	12.10	760
Nehetman	3	2400	2.38	98	12.38	720

* The same year, later, this horse carried off yet another prize, having covered a distance of 2400 m in 2 min. 34[4] sec.

med that the lower intensity of blood formation is fully compensated by an exceptionally strong heart which, with a lower concentration of the oxygen per unit of blood volume, insures a more rapid flow of the blood. Such cases only stress the need for a more complex physiological study of the animals.

Having proved in one of our previous papers the hereditary nature of the differences of the blood composition and in view of the simplicity of the methods of haematological study, we may even at the stage of our studies advise the breeder to make use of haematological analyses in his work at the stud farm.

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GENETIK

HYBRIDISATIONSVERSUCHE ZWISCHEN DER KARAUSCHE UND DER KULTUR- UND WILDFORM DES KARPFENS

Von E. A. HOCHLINA

(Vorgelegt von I. I. Schmalhausen, Mitglied der Akademie, den 28. XI. 1940)

Die vorliegende Mitteilung behandelt die Morphometrie, die Wachstumsintensität und die Lebensfähigkeit der von uns gewonnenen Bastarde. Die Resultate der zyto- und histologischen Bearbeitung werden in der nachfolgenden Mitteilung gegeben.

Die Versuche wurden im Jahre 1939 und 1940 in der Nikolsky Fischerei (Leningrader Gebiet) durchgeführt. Durch künstliche Befruchtung wurden folgende Kreuzungen erzielt:

1) und 2) 1939, 1940. ♀ japanische Goldkarausche [*Carassius auratus* L., Kulturform] × ♂ Teichkarpfen (*Cyprinus carpio* L.). Die Nachkommenschaft wird fernerhin als «Goldhybride» bezeichnet. 3) 1939. (Kontrolle) ♀ Taparavani Karpfen — ♂ Teichkarpfen. 4) 1940. (Kontrolle) ♀ und ♂ japanische Goldkarauschen. 5) 1939. ♀ Hybrid F_1 (Taparavani Karpfen × Teichkarpfen) × ♂ Seekarausche (*Carassius carassius* L.). Die Nachkommenschaft wird fernerhin als «dreifache» Hybride bezeichnet. 6) und 7) 1939, 1940. ♀ Hybrid F_1 (Karpfen-Karausche, durch Kreuzung der Seekarausche mit dem Karpfen entstanden) × ♂ Teichkarpfen. 8) 1939. Dasselbe ♀ × ♂ Wolga-Kaspien wilder Karpfen. 9) 1940. Dasselbe ♀ × ♂ Seekarausche.

In folge der vollständigen Sterilität der männlichen Karpfen-Karauschen, sind die Einkreuzungsversuche der Karpfen-Karauschen misslungen. Ungeachtet der stark ausgesprochenen bräutlichen Kleidung der Männlein, sonderte sich nur in grossen Mengen eine klebrige, aber durchsichtige Flüssigkeit ab, die gar keine Spermien enthielt; Befruchtung war nicht vollzogen worden. In der Literatur gibt es Hinweise in bezug auf die Fruchtbarkeit der männlichen Karpfen-Karauschen. Ihre Sterilität ist offensichtlich auf die Kreuzungsrichtungen und die Eigenschaften der allerersten Erzeuger zurückzuführen.

Die weiblichen Karpfen-Karauschen haben einen Laich abgelegt, der äusserlich ganz normal und befruchtungsfähig war; aber seine Lebensfähigkeit (6., 7., 8. und 9. Kreuzungen) hat sich als stark vermindert gezeigt. Sogar am letzten Inkubationstage des Laiches (die Inkubation geschah auf Marlerahmen) wurde ein grosser Verlust beobachtet, obgleich der Karpfen- und der Karauschenlaich um diese Jahreszeit beinahe keinen Verlust zeigt. Für die ganze Inkubationsperiode betrug der Verlust über 95%. Wenige aus dem verbliebenen lebendigen Laich ausschlüpfende junge Tiere (80 Stück in der 6-ten Kreuzung, 100—in der 8-ten, 5—in der 7-ten und 4—in der 9-ten) sterben weiter ab, wobei eine grosse Menge missgestalteter Jungtiere beobachtet wurde. Im Jahre 1939 gelang es keiner der jungen Tiere bis zum Alter von

zwei Monaten aufzuziehen; 1940 ist eines (7-te Kreuzung) am Leben geblieben. Es hat den Anschein, als ob die Ursache der mangelnden Lebensfähigkeit der sehr grossen Mehrzahl der Keime in einer Störung des Chromosomenbestandes der weiblichen Karpfen-Karauschen liegt. Eine zytologische Analyse wird uns in Stand setzen, diese Vermutung zu prüfen.

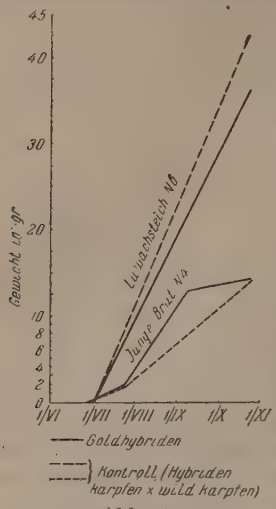

Abb. 1.

In der 1-ten, 2-ten und 5-ten Kreuzung entwickelte sich der Laich normal. Auf den Rahmen mit Goldhybriden betrug der Verlust im ganzen ungefähr 22% (1939), auf denen mit dreifachen Hybriden — ungefähr 27.5%. Die Kontrollrahmen (3-te Kreuzung) ergaben nur 10—15% Verlust, was vielleicht auf einen grösseren Prozentsatz der Befruchtung zurückzuführen ist, als in den Hybridkreuzungen. Die Angaben auf die Aufzucht des Materials in Sommerteichen bestätigen eine höhere Lebensfähigkeit der Goldhybriden und dreifachen Hybriden im Vergleich mit den Kontrollhybriden: Karpfen × wilder Karpfen. Im Jahre 1939 erlitten die Goldhybriden 25.8% Verlust für den Sommer; die Kontrollhybriden 94%; die dreifachen Hybriden 26.8%; Kontrollhybriden 76% (bei gleichzeitigem Besatz und gleichem Besatzgewicht).

Die Angaben bezüglich der Überwinterung des Hybridenmaterials sind von noch grösserem Interesse. Im Laufe des sehr rauhen Winters 1939—1940, ergaben die dreifachen Hybriden 35.3% Verlust, die Goldhybriden 70%, die Teichkarpfen dagegen (in demselben Teich — Kontrolltiere) waren gänzlich abgestorben (100%), trotz der grossen Dimensionen und des grossen Gewichts. Alle diese Angaben sprechen für eine hohe Lebensfähigkeit der dreifachen und der Goldhybriden und, ohne Zweifel, für eine starke Winterwiderstandsfähigkeit im Vergleich mit Kulturkarpfen.

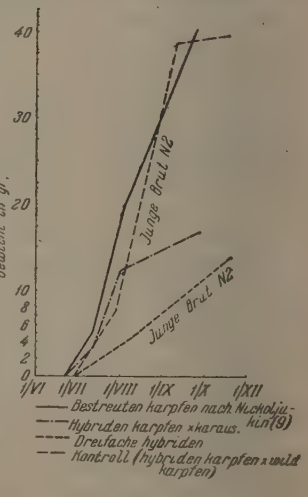

Abb. 2.

Zum Zweck des Studiums der Wachstumsintensität und der Morphologie der betreffenden Fische, wurden Proben von einsömmerigen Gold- und dreifachen Hybriden insgesamt mit den Kontrolltieren, ebenfalls Proben von japanischen Goldkarauschen, die bei den Kontrollabfischungen im Sommer und bei den Herbstabfischungen genommen wurden, eingehend analysiert. Ausser Gewicht und Körperlänge, wurde eine ganze Reihe von äusserlichen und anatomischen Mess- und Zahlmerkmalen untersucht. Die sämtlichen Messmerkmale wurden in Zusammenhang mit der gesamten Körperlänge gebracht (Angaben in %); ausserdem, wurde die Körperlänge mit der Körperhöhe verglichen (Index l/H), ebenfalls die Stirnbreite mit der Kopflänge (in %). Solche Merkmale wie Gewicht, Körperlänge und der Index l/H erlauben den ökonomischen Wert der Hybriden zu beurteilen. Die anderen gelten hauptsächlich für die Systematik.

G o l d h y b r i d e n. D 15—20, am häufigsten 17; A III+5, *V* 5 — 10, häufiger 9, *l. l.* $30\frac{5-6}{6}38$, häufiger $32\ \frac{5}{6}\ 33$, Kiemenborsten 30 — 37. Index $l/H = 2.38 \pm 0.02$. Die Körperfärbung ist schwachgoldig, hell,

die Flossen rötlich. Was die Körperform betrifft, besonders den hohen Rücken (hinter dem Kopf bildet der Rücken einen Höcker), erinnern diese Hybriden an die japanischen Goldkarauschen. Die Schwanzflosse ist langlappig mit einem tiefen Einschnitt zwischen den Gliedern. Die Rückenflosse ist hoch, vorn wellenartig gebogen. Sie errinnert durch ihre Form an die Rückenflosse des Karpfens, durch ihre Höhe aber kommt sie der der Karausche nahe. Der Körper ist dicht mit grossen Schuppen bedeckt, die Seitenlinie ist glatt. Der Kopf ist etwas verlängert, Augen sind gross.

Dreifache Hybriden. D 15—20, am häufigsten 18; A III + + 5 — 6; *V* 8 — 10, häufiger 9, *l. l.* 34 $\frac{6-7}{6-8}$ 38, häufiger 36 $\frac{6}{7}$ 37, Kiemenborsten 24 — 30. Index $l/H = 2.75 \pm 0.01$. Die Färbung des Körpers und der Flossen ist dunkelbraun mit einem schwachen goldenen Ton. Hinsichtlich der Rückenhöhe unterscheiden sie sich sehr von der Goldhybride, und, dem Anscheine nach, auch vom Karpfen, indem sie nur ein kleinwenig hochrückiger sind als die Kontrollhybriden. Kleiner Kopf, kleine Augen (Ähnlichkeit mit der Seekarausche). Der Körper ist dicht mit Schuppen bedeckt, die aber kleiner sind als bei den Goldhybriden. Die Rückenflosse steht der Form nach näher der des Karpfens, errinnert aber in bezug auf ihre Höhe an die der Karausche. Was die Strahlenzahl an der Rücken- und der Afterflossen betrifft, stehen diese Hybriden den Seekarauschen näher als den Karpfen, besonders charakteristisch ist die nicht seltene Karauschen Formel: A III+6.

Die Goldhybriden, wie auch die dreifachen Hybriden, besitzen nur ein Paar Barthaare, die bedeutend kürzer sind als die des Karpfens. Von den anatomischen Merkmalen ist die Zahl der Kiemenborsten am I. Kiemenbogen neben der Zahl und den Formeln der Schlundzähne von besonderem Interesse. Die Goldhybriden besitzen eine viel grössere Zahl Kiemenborsten, als die dreifachen Hybriden. Bei den grössten einsömmerigen Fischen gibt es am innern Rand der Kiemen durchschnittlich 35.22 ± 0.39 Kiemenborsten bei den Goldhybriden, bei den dreifachen Hybriden 26.25 ± 0.76. Dies entspricht dem Unterschied zwischen den Karauschen; die japanische Goldkarausche besitzt eine viel grössere Anzahl Kiemenborsten, als die Seekarausche (⁹). Der Mittelwert der Kiemenborsten bei den einsömmerigen Goldkarauschen betrug, nach unseren Angaben, 39.70 ± 0.45.

Die Formel der Schlundzähne variiert stark bei den beiden Gruppen, insbesondere bei den dreifachen Hybriden. Die Formel 1.4—4.1 spricht zugunsten einer abwechselnden Vererbung der Schlundzähne in den beiden Kreuzungen. Andererseits, ist eine gewisse Matroklinie bemerkbar. Bei den dreifachen Hybriden sind zuweilen die Formeln 1.3—4, 1.4—3.1, 1.1.4—3.1, u. a. anzutreffen (obgleich für die Kultur-, wie auch für die Wildform des Karpfens die Formel 1.1.3—3.1.1 mehr charakteristisch ist; 1.1.3—3.1 oder 1.3—3.1.1 sind wohl auch zuweilen anzutreffen). Bei den Goldhybriden aber wurde die Karpfenformel niemals beobachtet: die Goldkarausche war der weibliche Erzeuger. Dementsprechend finden wir die reine Karauschenformel (4—4) bei den Goldhybriden öfter als bei den dreifachen.

Zusammenfassend, kann man über die Vererbung der mannigfachen Merkmale von den Hybriden folgendes feststellen. Die Goldhybriden stehen der japanischen Goldkarausche nahe, insofern es um die Rückenhöhe, um die Strahlenzahl in der Rückenflosse, um die Schuppenzahl längs der Seitenlinie und der Zahl der Kiemenborsten handelt. Die dreifachen Hybriden sehen der Seekarausche ähnlich in bezug auf die Kopfgrösse («kleinköpfige»), die Strahlenzahl in D- und A-Flossen, und die Zahl der Schuppen längs der Seitenlinie. Die Breite der Stirn, die Höhe der Flossen D und A, die Schlundzähne und Barthaare (ein Paar) zeigen bei den beiden Hybriden einen abwechselnden Charakter.

Im ersten Sommer wachsen die Goldhybriden sehr schnell (Abb. 1). Der Vergleich wurde zwischen ihnen und den Hybriden der Kultur- mit der Wildform des Karpfens durchgeführt; wenn wir berücksichtigen, dass die letzten, dank der Heterosis, schneller wachsen als die Karpfen, dürfen wir mit der grössten Sicherheit feststellen, dass die Wachstumsintensität der Goldhybriden der des Karpfens nicht nachsteht. Wenn wir ferner in Rechnung ziehen, dass der Mangel an Nahrung eine allgemeine Verlangsamung des Wuchses zur Folge hatte, müssen wir sogar annehmen, dass sie die Hybriden der Kultur- mit der Wildform des Karpfens überholen. Dies lässt vermuten, dass die Heterosis bei den Goldhybriden hauptsächlich auf die frühe Lebensperiode fällt [ähnlich den Hybriden der Kultur- mit der Wildform des Karpfens]. Die japanischen Goldkarauschen bleiben in bezug auf Gewicht hinter den Goldhybriden bedeutend zurück. Bei der 1940 durchgeführten gemeinschaftlichen Aufzucht (Kreuzungen 2 und 4) im Lauf eines Monats (seit 11/VII bis zum 14/VIII 1940) erreichten die Goldhybriden ein durchschnittliches Gewicht von 4.55 g, während die Karauschen, in demselben Zeitraum, unter denselben Aufzuchtsbedingungen und dem gleichen mit den Hybriden Anfangsgewicht, nur 1.62 g erreichten. Die getrennte Aufzucht 1939 ergab ähnliche Resultate.

Die Wachstumsintensität der dreifachen Hybriden bleibt hinter der der Hybriden der Kultur- mit der Wildform des Karpfens bedeutend zurück, und, demnach, auch hinter der der Karpfen (Abb. 2). Ein ähnliches Zurückbleiben wurde von Nikoljukin bei den einsömmerigen Karpfen-Karauschen im Vergleich mit dem Karpfen festgestellt.

Schlussfolgerung. Unterschiede zwischen den Karauschen—der japanische Goldkarausche (*Carassius auratus* L.) und der Seekarausche (*Carassius carassius* L.)—haben sich ebenfalls an den untersuchten Hybriden gezeigt. Goldhybriden (*Carassius auratus*×Karpfen) und dreifache Hybriden [(Kulturform×Wildform des Karpfens)×*Carassius carassius* L.] unterscheiden sich in bezug auf ihre Wachstumsintensität, wie auch nach ihren morphologischen Merkmalen, stark von einander. Neben einem starken Einfluss der Karauschenart auf die Hybriden, ist bei den letzteren eine intermediäre, wie auch eine mütterliche Vererbung von etlichen Merkmalen beobachtet. Eine überaus hohe Lebensfähigkeit der Hybriden, im Vergleich mit dem Karpfen, ist die Folge der erblichen Übertragung der allgemein bekannten Widerstandsfähigkeit der Karausche; auch ist das Vorkommen von der Heterosis möglich (unzweifelhaften bei den Goldhybriden). Die erste Generation der Hybriden des Karpfens mit der Karausche hat für sich eine bestimmte wirtschaftliche Bedeutung, als ein Material für die Bevölkerung der Wasserbehälter, die, aus diesem oder jenem Grunde, für die Karpfenzucht nicht geneigt sind. Unsere Aufgabe schliesst aber die Gewinnung von fruchtbaren Hybriden der Karpfen-Karauschen ein, und die Einkreuzungen des Karpfens mit *Carassius auratus* L., die Gewinnung von dreifachen Hybriden, u. s. w., können, in dieser Hinsicht, eine bestimmte Rolle spielen. Ein eingehendes zyto- und histologisches Studium der aus diesen Kreuzungen entstandenen Keimen wird uns ermöglichen die Ursachen der Sterilität der männlichen Karpfen-Karauschen festzustellen und dieselben, wenn auch teilweise, zu entfernen.

Der experimentelle Teil der gegenwärtigen Mitteilung, die Bearbeitung des gewonnenen Materials und die Redaktion des Artikels wurden unter der unmittelbaren Leitung von D-r V. S. Kirpitschnikov durchgeführt, und ich ergreife diese Gelegenheit, um ihm meinen Dank auszudrücken.

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PLANT PHYSIOLOGY

SPRAYING WITH MICROELEMENTS AS A METHOD OF INCREASING THE RATE OF PHOTOSYNTHESIS

By A. A. RICHTER, Member of the Academy, and N. G. VASSILIEVA

Among the main problems facing the socialist plant industry is that of increasing the productive power of crops. To transform the cultivated plants into highly efficient organisms working at an increased rate, is the aim of a series of agrotechnical measures which set up what we call «normal» conditions for the nutrition and thereby for growth and development of the vegetable organism concerned. But it is essential also to add to this a number of special stimuli which enable the organism to raise its creative functions far above the usual level. Such an intensification of the assimilative and photosynthetic work of our crops is the goal to which our practical effort backed by the experiment should be directed.

Among the stimuli urging the plant to higher efficiency is the one generated by the introduction into the nutrient medium of the so-called microelements. These elements, which are not usually included into the composition of a normal nutrient solution, can, if introduced there in slightest concentrations, bring about an unmistakable increase in yield (if applied in any appreciable concentration they are very poisonous for plants). Such are boron, manganese, zink, copper etc. which have now become notorious in the physiology of plant nutrition.

It was not, of course, the concern of those working in this laboratory to repeat the numerous investigations which had already put beyond all doubt the important rôle of microelements in the life activity and productive ability of plants. The task they set themselves was to develop practical methods, by which a direct control of the assimilation apparatus in plants would become possible with a minimum expense of the rather costly material. The basic method in their attempts and one which can be used in the field, was spraying the leaves directly with weak solutions of various microelements.

This method was tried on sunflower, horse-beans, kok-saghyz, hydrargea and perilla. The plants were grown in beds or in pots with a supply of complete fertilizer (N, P, K). Spraying was done by means of an atomizer. The solutions so applied were $ZnSO_4 \cdot 7H_2O$; $KMnO_4$, KI and H_3BO_3. The rate of photosynthesis was determined with the aid of Richter's adsorber on whole leaves cut off from the plant or on their portions of a standard size. The standartization of the leaf areas tested secured an almost complete coincidence of parallel determinations. Electric bulbs, used as a source of energy, gave an illumination intensity of about 20000 luxes.

Every test involved 2 experimental objects, 2 controls sprayed with water and 2 devices for estimation of the carbon dioxide content in the current of air passing at a rate of 30 l per hour. The duration of each test was 20 min. when the rate of photosynthesis was measured and 1 hour when the rate of

respiration was measured. Hereafter are presented some data characterizing the process.

Test 1. July 10, 1940. Leaves of horse-beans (*Vicia faba*); spraying with a 0.02% solution of KI was done in the evening preceding the day of testing. Assimilation of carbon dioxide per 1 dm^2 per hour: control 6.5 mg, or 100; experiment 8.5 mg, or 130.5. Increase in the rate of photosynthesis, 30.5%.

Test 3. Leaves of *Vicia faba* sprayed with a 0.02% solution of H_3BO_3 in the evening July 5. Measurements made July 6, 8 and 10.

Assimilation of Carbon Dioxide per 1 dm^2 per hour

	Control	Experiment
July 6	6.2 mg (100)	4.55 mg (733)
July 8	5.25 mg (100)	7.15 mg (136)
July 10	6.75 mg (100)	8.0 mg (118.5)

Reduction of the photosynthetic activity on the first day after spraying was followed by a strong increase.

Test 6. Sunflower leaves sprayed with solutions of $KMnO_4$, 0.000632 Mol and 0.001264 Mol, the day before. Assimilation of carbon dioxide per 1 dm^2 per hour: Weak solution: control 7.83 mg (100), experiment 8.8 mg (112.3). Strong solution: control 8.1 mg (100), experiment 10.15 mg (125.3). Increasing the concentration from 0.01 to 0.02% has brought about an increase in the rate of assimilation.

Test 8. Leaves of kok-saghyz sprayed with solutions of $ZnSO_4 \cdot 7H_2O$, 0.000174 Mol and 0.00000348 Mol, on August 6. Assimilation measured August 7 and 13.

The strong increase in the rate of photosynthesis caused by spraying the leaves with zink salts lasted for about a week (6 VIII—13 VIII).

Assimilation of Carbon Dioxide per 1 dm^2 per hour

		Control	Experiment
Weak solution	Aug. 7	5.4 mg (100)	10.3 mg (190.7)
	Aug. 13	4.8 mg (100)	10.59 mg (220.2)
Strong solution	Aug. 7		8.76 mg (162.2)
	Aug. 13		5.66 mg (117.6)

Test 10. Perilla leaves sprayed with a 0.000348 Mol solution of $ZnSO_4 \cdot 7H_2O$ the day before testing. Assimilation of carbon dioxide per 1 dm^2 per hour: Control 9.9 mg, making allowance for respiration 11.8 mg (100); experiment 12.37 and 17.05 respectively (144).

Test 11. Hydrargea leaves treated similarly. Assimilation of carbon dioxide per 1 dm^2 per hour:

1) On the next day. Control (allowance for respiration) 10.08 mg (100); experiment (allowance for respiration) 15.41 mg (152.8).

2) After 10 days. Control (allowance for respiration) 4.39 mg (100); experiment (allowance for respiration) 5.33 mg (121.3).

These data have been taken from an array of similar data obtained in other tests which have all furnished evidence that very weak solutions of microelements sprayed one time on the leaves will increase their assimilation work, that is make them more heavily stored with assimilates, and that this stimulative effect will not vanish for several days, only gradually decreasing in strength. Now the experiments are to be continued in the experimental plots in the field, and spraying will be done several times at different periods of development.

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PLANT PHYSIOLOGY

DEVERNALIZATION OF VERNALIZED WINTER WHEATS

By A. K. EFEIKIN

(Communicated by A. A. Richter, Member of the Academy, 24. XII. 1940)

It had been established by us previously (1) that vernalized plants of winter wheat could be devernalized by keeping them for the first 6 days following planting at as high a temperature as 27—33° C. Now in the present work we have set before us the task to follow up this action of high temperature in time and to find out the shortest interval sufficient to bring about the reversion.

As before, we have carried on our experiment with variety Durable. The seeds were divided into seven portions. The first consisted of unvernalized seeds and was to serve as control. All the others included seeds vernalized at 0—2° C during 45 days. Among them the seeds of the 2nd portion were left without further treatment. But the remaining 5 portions just after vernalization were subjected to the action of high temperature, 34—35°C, for various lengths of time. The 3rd portion was so treated during 1 day, the 4th, 5th, 6th and 7th during 2, 3, 4 and 5 days, respectively. An electric thermostat provided with a thermoregulator was used for that purpose. The seeds were kept there in a Petri dish with filter paper under and over them. Drying of the filter paper was offset by wetting it with water of the same temperature. Under these conditions some growth, though a very slow one, was made by the seedlings. This treatment was followed by planting. It was, of course, essential to plant all the tested seeds on one day. But their stay in the thermostat had been a varying term, while the length of the vernalization interval had been equal for all seeds. Still on one day, May 14, they were planted. This had been made possible by starting the vernalization process at different dates. So the 7th portion of seeds had been subjected to vernalization on March 24; the 6th on March 25; the 5th on March 26; the 4th on March 27; the 3rd on March 28 and the 2nd on March 29. Accordingly, they had been set in the thermostat on May 9, 10, 11, 12 and 13.

Planting was done in small clay pots filled with black soil. Every pot received 10 grains at first, but when the seedlings had appeared above the surface only 7 plants out of the 10 were left to grow further. As the pots were too small for this number of plants, nutrient mixture was added several times during the growing period, and the plants all along had a perfectly sound and normal appearance. The experiment was carried on in the open and lasted until September 7. The results of observation are set forth in the table, and the condition of the plants by the end of the experiment is shown in Fig. 1.

Effect of Presowing Treatment with High Temperature on Winter Wheat Seedlings, Variety Durable

Pot No.	Experimental sets	Day of planting	Day of emergence from the soil	Day of earing	Number of plants in the set	Number of those which had formed ears	% of those which had failed to ear
1	Unvernalized	May 14	May 19	—	21	—	100.0
2	45-day vernalization .	May 14	May 18	July 11	21	21	0
3	45-day vernalization+1 day of high temperature	May 14	May 17	July 30	21	7	66.7
4	45-day vernalization+2 days of high temperature	May 14	May 17	July 24	21	2	90.5
5	45-day vernalization+3 days of high temperature	May 14	May 17	Aug. 1	21	1	95.2
6	45-day vernalization+4 days of high temperature	May 14	May 17	Aug. 12	21	1	95.2
7	45-day vernalization+5 days of high temperature	May 14	—	—	21	—	100.0

What is brought out by the tabulated data is this:

1) Among the vernalized plants those which had not been treated with high temperature (set 2) had all formed ears. Into ear had come their main stems and partly also their lateral shoots.

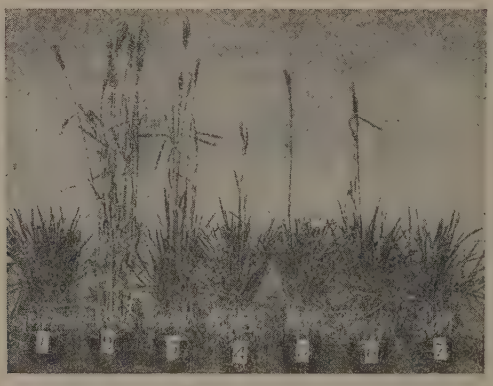

Fig. 1. Experimental plants 98—100 days after they appeared above ground.

2) The plants kept for 5 days in the thermostat (set 7) had, like the unvernalized control, failed to form ears. In the growing cones of these plants there was no signs of spikelet differentiation. Obviously, they had been devernalized by the action of high temperature.

3) The ratio of earless plants increased with the length of treatment to become 100% in the case of plants kept for 5 days in the thermostat. At first the increase is rapid, but afterwards expires gradually. If the number of earless plants is taken as a quantitative expression of the action of high temperature, this action can easily be represented graphically, as it is done in Fig. 2,

where the ratio of earless plants is plotted against the duration of treatment. The curve thus obtained has a shape characteristic of curves expressing the dependence of various physiological processes upon an increasing influence of external factors. So run e. g. the curves bringing out the relation of photosynthesis to light, CO_2 etc. In other words, the process of devernalization is governed by the general laws underlying the physiological processes.

The fact that plants refusing to ear can arise from vernalized seeds has already been reported by Lebedev and Sergeev ([2], [3]). These authors had slightly dried their seeds after vernalization and kept them for a long time before sowing. Strongly tillering plants bearing no ears had grown up from these seeds. Their interpretation of the phenomenon differed from that evolved somewhat later by Gregory and Purvis ([4]), who came next to deal with the problem, and was that the initial vernalized growing point had died off and that the plants had regenerated out of unvernalized lateral buds. This explanation is impossible in our case, for we kept our plants under systematic and careful observation, giving special attention to the development of the growing cone of the embryo bud, and not in a single case did we find the growing point to have died away or developed abnormally. Besides, it is based on the theory which supposes the phasic changes to be strictly localized in the meristem of the growing cone and to be irreversible, which is inconsistent with an array of facts, as has been shown by us in an earlier paper ([5]). The incorrectness of such explanation is also proved by the fact that in the plants grown from untreated vernalized seeds (set 2) the lateral shoots have formed ears as well as the main stems. This will mean that the influence of vernalization is not confined to the growing point of the embryo bud, but extends also over the lateral buds. Nothing therefore on ground of this interpretation could have prevented our plants in set 7, as well as the plants in the experiment of Lebedev and Sergeev, to form ears, what as a matter of fact they did not.

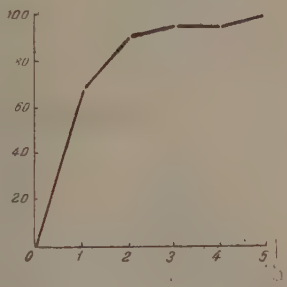

Fig. 2. High temperature treatment of wheat seeds; its effect in relation to its length.

The data reported here have furnished new evidence in support of the statement expressed by us previously ([1]): that the vernalization process is a reversible one.

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PHYTOPATHOLOGY

DIAGNOSIS OF VIRUS DISEASES OF CEREALS

By V. K. ZAZHURILO and G. M. SITNIKOVA

(Communicated by A. A. Richter, Member of the Academy, 21. XI. 1940)

The identification of a virus disease in plants is a matter of considerable difficulty. A number of complicated determinations requiring a special laboratory equipment are often involved. Naturally enough, there has been a constant search for more practical methods available to a wider circle of workers. Of these, two, the anatomical ([6],[7]) and the serological ([1],[9]) appear to be the most simple. The latter is now successfully used in practice. But the difficulty with it is to obtain the specific serum when the virus is not transmitted mechanically through the juice. This is exactly the case with the winter wheat mosaic and zakuklivanie, two virus diseases belonging in the group of yellows. Our aim therefore has been to base a proper diagnosis of these diseases on some microscopically detectable modifications in the anatomy and cytology of the affected plants ([2],[5]), concerning the winter wheat mosaic to which our attention has been mainly devoted. It may be remarked that its vector, *Deltocephalus striatus* L., does not transmit the virus of zakuklivanie ([8]), thus making it possible experimentally to obtain plants affected with the virus of winter wheat mosaic alone.

To microscopic investigations the following plants, both sound and diseased, have been subjected: 1) wheats—*Tr. vulgare*, *Tr. durum*, *Tr. spelta*, *Tr. monococcum*, *Tr. dicoccum*, *Tr. persicum*, *Tr. compactum*, *Tr. polonicum*, *Tr. Timopheevi*; 2) rye, 3) barley, 4) oats, 5) millet, 6) mohar, 7) rice. Fresh as well as fixed materials (5—10% formalin, 96° alcohol, liquors of Navashin, Régaud and Zenker) have been investigated. Under the microscope were brought mainly transverse and longitudinal leaf-sections made by hand or with a microtome. The epiderm was also examined after being stripped off from the under and upper side of the leaf. The paraffin cuts were stained with haematoxylin according to Heidenhain; the cuts made by hand, with acid fuchsin.

The most general symptom of the disease is a phloem affected with necrosis. The latter is easily detectable on the cross-sections of leaves in the regions of light-green tissue (Fig. 1). The necrotic cells of the phloem are filled up with a yellow substance; the cell walls are also impregnated with it and sometimes it penetrates into the intercellular spaces. By the fluoroglucin and hydrochloric acid the necrosed areas are stained in various shades of red, which is evidence for the presence of wound gummy. In the absence of sharply expressed necroses only the walls of the phloem cells are stained by the fluoroglucin, while the contents remain uncoloured. It will be obvious that

in early phases of necrosis the cell walls alone are impregnated with wound gummy, which only later comes to fill up the entire cavity of the dying cell.

The cereal mosaic is in the first place a disease of the phloem, but, unlike the typical yellows, it also affects the chlorophyllian tissues and by so doing becomes responsible for the underdevelopment and reduction of plastids. The apparatus of assimilation is disturbed, first of all, in those regions of the mesophyll which adjoin the phloem. Here, even at the early stages of the disease, the chloroplasts in the cell are fewer and smaller compared to the sound regions. By and by the degeneration of plastids spreads throughout the thickness of the mesophyll, and the vascular bundle is eventually bounded on the sides and underneath by the affected tissue, while the cells contiguous to the xylem retain their normal quantity of chloroplasts. In the case of a strong affection the degeneration of plastids takes place in this part of the mesophyll, too.

In the cereals with a coronal type of chlorophyllian parenchyma (millet and others) only the cells inside the inner circle of the parenchyma, the one adjoining the vascular bundle, are affected at the early phases of the disease. This is betrayed by a characteristic lightening along the path of the nerves. Later on the affection involves the outer circle of the parenchyma. So the anatomical picture of the disease affords indirect evidence of a gradual spreading of the virus from the phloem to the cells of the mesophyll.

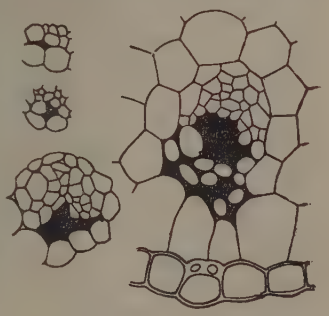

Fig. 1. Necrotic phloem in leaf's vascular bundles: on the right, next to the phloem, the parenchymal cells affected with necrosis.

Vascular inclusions were not observed in the cells in all cases. The most often they occurred in old leaves strongly affected with mosaic. Their formation was confined to the cells of the mesophyll, and in no other tissue of the leaf could we detect them. The significance of vascular inclusions with regard to diagnosis and etiology, which many authors have supposed them to have, needs revision, for they have recently been discovered by Sukhov ([8]) in old leaves of sound plants. Apparently, they are as well a product of secretion by a sick as by an ageing cell ([5,7]). It may be remarked, however, that the same plant does not respond in the same way to the affection by all viruses, but rather forms more vacuolar inclusions in one case than in another and sometimes does not form them at all ([10,11]). It would be unjust therefore to deny any diagnostic importance to these cell inclusions.

No protein crystals were ever found by us in the plants infected with the virus by *Deltocephalus striatus*.

Comparing the results of our investigation with the data on other virus diseases of cereals, available in literature, we arrive at the following conclusions: 1) Wheat viruses I and IA of McKinney differ both from the virus of winter wheat mosaic and from that of zakuklivanie in leaving the phloem unaffected. 2) Protein crystals that appear in the cells of plant's portions affected with zakuklivanie are not formed in the case of winter wheat mosaic*.

So a differential diagnosis of cereal virus diseases becomes possible on the basis of anatomical and cytological characters:

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* The absence of crystalline inclusions of protein in the cells of plants affected with winter wheat mosaic has been confirmed by Sukhov, who found such plants in the vicinity of Moscow in 1940.

Name of virus	Vacuolar bodies	Necrosis of phloem	Protein crystals
Wheat virus I, McKinney	+ + +	—	—
Wheat virus IA, McKinney	+	—	—
Winter wheat mosaic, Zazhurilo and Sitnikova	+ + +	+ + +	—
Zakuklivanie, Sukhov and Vovk	+ + +	+ + +	+ + +

By means of the anatomical method each of these diseases can be identified properly, whereas the external appearance of affected plants is no ground for correct judgment, as different viruses can be responsible for many similar external symptoms in cereals. It is by using this method that we were able, as early as 1939, to state the occurrence of winter wheat mosaic virus in the provinces of Sumy (from the specimens supplied by N. V. Broiakovsky) and Krasnodar (from the specimens sent by P. G. Ruzinov). And in 1940 it served for identification of the same virus in a number of provinces in the European part of the USSR. The investigations were carried on at this laboratory on behalf of the Committee for Grain Crop Variety Tests, who had sent the specimens of affected plants from their experimental plots for microscopic examination. The virus was found to be spread in the following provinces: Moguilew, Riazan, Orel, Kursk, Voronezh, Saratov, Kuybyshev, Autonomous Soviet Socialist Republic of Volga Germans, Sumy, Kharkov, Zaporozhie, Rostov and Krasnodar. Not in a single case did we find protein crystals in the specimens of affected plants from those regions, while we always found them in the specimens from Siberia, where the virus of zakuklivanie is known to be widespread.

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PHYTOPATHOLOGY

BIOCHEMICAL MODIFICATIONS IN THE CEREALS AFFECTED WITH THE VIRUS OF WINTER WHEAT MOSAIC

By N. A. RIAKHOVSKY and A. L. FEDULAEV

(Communicated by A. A. Richter, Member of the Academy, 15. XII. 1940)

The symptoms of the disease, for which the virus of winter wheat mosaic is responsible, have been thoroughly studied by Zazhurilo and co-workers ([1],[2],[3]), but the biochemical modifications taking place in the plants so affected have been mentioned only occasionally, and the present work is the first attempt to explore more fully this subject.

The materials involved in the investigation were: 1) fresh specimens of millet (variety *Aureum* 198) collected while at the stage of filling in the fields of the Ramon breeding station, August 3, 1940; 2) herbarium specimens of winter wheat (0237, the Saratov station), collected by Zazhurilo in the region of Talovaia, June 1937; and 3) specimens of winter wheat (0237, the Saratov station, tillering stage) and oats (Pobeda, stage of culm formation) fixed in an autoclave after they had been infected, by means of *Deltocephalus striatella*, with the virus in question([3]). Only the leaves from these specimens were taken for analysis and not the whole plants, in view of the experience gained by Rischkov, Vorobieva and Gromyko ([4]).

Of the carbohydrates, mono- and disaccharides were estimated iodometrically according to Issekutzu; and starch with hemicellulose as a total. The results obtained are summarized in Table 1 (% in abs. dry matter).

These data show that in response to infection both the mono- and disaccharides have strongly increased in amount. And so has also the sum of starch and hemicellulose in the case of millet. But in the case of oats and wheat a decrease of this sum is observed, probably for the following reason. Millet was collected in the morning, oats and winter wheat, in the afternoon.

Table 1

Plant	Place	Material	Sugars			Starch and hemicellulose together
			Monosaccharides	Disaccharides	Their sum	
Millet	Collected in the field	Affected	12.20	18.30	30.50	6.70
	Do	Sound	5.30	2.28	7.58	4.90
Oats	Artificially infected	Affected	5.36	14.14	19.50	26.06
		Sound	2.63	9.22	11.85	28.13
Winter wheat	Do	Affected	6.71	13.89	20.60	26.23
		Sound	3.74	6.47	10.21	27.40

If the outflow of assimilates from the leaves of affected plants ([1]) and the stronger development of sound plants is now taken into consideration, it will be an easy matter to explain the deviation. Owing to a more energetic photosynthesis, the sound plants in the course of day increase more in starch than the affected. This factor was excluded in the case of millet.

Total N, protein N and non-protein N were estimated, as well as total and mineral P. The total N was determined iodometrically, the protein N, according to Barnstein. The total phosphorus was determined according to Pamberton, while the mineral one was determined in the residue left after the material had been treated with ether. The data obtained are brought together in Table 2 (% in abs. dry matter).

Table 2

Plant	Place	Material	Nitrogen			Phosphorus	
			Total	Protein	Non-protein	Total	Mineral
Millet	Collected in the field	Affected	2.63	1.86	0.057	0.99	0.41
	Do	Sound	3.04	2.44	0.069	1.01	0.39
Oats	Artificially infected	Affected	3.03	—	—	1.30	—
		Sound	3.74	—	—	1.46	—
Winter wheat	Do	Affected	3.28	1.48	—	1.43	—
		Sound	3.48	1.96	—	1.51	—
Winter wheat	Collected in the field	Affected	2.71	1.78	—	0.60	0.26
	Do	Sound	3.09	2.63	—	0.70	0.27

The content of both total and protein nitrogen is somewhat lower in affected plants than in sound. A slight decrease in response to infection (millet) is shown also by the non-protein nitrogen. That the plants affected with the winter wheat mosaic have been found to decrease in nitrogen, is a very interesting observation indeed, because, as has been stated by Rischkov ([5]), there are only very few indications for such a response of plants affected with virus diseases, and the virus nature in our case has been proved for oats and one of the winter wheat specimens experimentally.

As to the C/N ratio, the diseased plants differ much from the sound. In the case of millet this ratio is 4.1 in sound plants and 14.1 in the affected; for oats the respective figures are 10.6 and 15.0; for wheat, 10.8 and 14.1.

In a word, this and several other characters (lesion of phloem, transmission through the insects) are evidence of the disease studied being very near to the yellows. And again there are other characters, as the mosaic pattern, absence of proliferation and antholysis in the flowers, which suggest to class it with the mosaics. It will be obvious that like zakuklivanie of cereals, this disease holds an intermediate place, now approaching the yellows, now the mosaics, according to the character considered.

The deviations in the content of P were not considerable, but still there was an unmistakable tendency towards a decrease in the affected leaves.

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GEOBOTANIK

ZUR GESCHICHTE DER INSELARTIGEN WÄLDER DES KUSTANAI GEBIETES

Von I. A. KRUPENIKOV

(Vorgelegt von A. A. Richter, Mitglied der Akademie, den 28. XI. 1940)

Im nordlichen Kasachstan im Bereich des Kustanai Gebietes befindet sich eine Reihe isoliert voneinander gelegener Fichtenwald-Massive und Birken-Espen Wälder, von denen der Fichtenwald in Naurzum den südlichen Vorposten der Holzvegetation des ebenen Teiles des Kasachstans bildet.

Bei Untersuchung der Böden des National Parkes von Naurzum entdeckten wir ein altes, 12 km nördlich vom Fichtenwald gelegenes Torflager. Im Torf wurden in verschiedener Tiefe Pollen, Reste von Rinde und Holz verschiedener Baumarten aufgefunden, darunter auch solche, die gegenwärtig nicht nur im Fichtenwald von Naurzum, sondern auch 300—500 km nördlich von ihm (Erle, Tanne) nicht vorkommen. Das Torfmoor hat eine Mächtigkeit von über einem Meter, und sein Untergrund wird durch oligozänen Sand gebildet. In der Tabelle führen wir die Analysenergebnisse des Pollens und des Zersetzungsgrades des Torfes* an.

Tiefe der Proben in cm	Grad der Zersetzung in %	Pollen in %				
		Fichte	Tanne	Birke	Erle	Weide
2— 10	35	30	—	24	—	46
10— 20	35	14	2	—	—	84
20— 30	40	5	2	30	3	60
40— 50	30	5	—	58	3	34
60 — 70	25	7	—	20	4	69
80— 90	40	1	—	90**	—	9
90—100	55	3	—	58	4	35
100—110	45	4	—	72	2	22

Die tiefstgelegenen Schichten des Torfmoors (100—110 und 90 — 100 cm) entsprechen wahrscheinlich der borealen Periode des Beginns der Nacheiszeit. Zu dieser Zeit wird das Klima etwas feuchter: es nimmt die Menge des Pollens der Erle und Weide zu; es verringert sich relativ die Menge des Birkenpollens; der Torf ist nach Ergebnissen der Bestimmung seiner botanischen Zusammensetzung ein Holz-Schilf-Riedtorf. Für Westeuropa war zu dieser Zeit Erle-Schilftorf charakteristisch. In dem Torfmoor von Naurzum wurde in der entsprechenden Tiefe unter Holzresten ebenfalls Erlenholz entdeckt.

Weiter (Tiefe 80—90 cm) findet der Übergang zur trockenen xerothermischen Zeit statt: die Erle verschwindet, die Fichte verschwindet fast vollkom-

* Die Pollenanalysen wurden im Laboratorium der Zentralen Torfstation durch den Analytiker Minkina ausgeführt.

** Birkenbaumstumpf. Starke Beimengung von Sand.

men; der Torf ist mineralisiert. Bei diesen Bedingungen blieb ein ganzer Birkenbaumstumpf erhalten, der in einer Tiefe von 80—90 cm entdeckt wurde. Augenscheinlich ging das Verwehen des Naurzum Sandes und die Bildung der Sandhügel in der xerothermischen Zeit vor sich. Die Torfprobe 80—90 cm. der Analyse entsprechend, eine grosse Beimengung von Sand enthält.

Eine Tiefe von 60—70 cm entspricht dem Beginn einer neuen Feuchtigkeitsperiode: wiederum erscheint die Erle, es nimmt die Pollenmenge der Fichte und Weide zu, der Grad der Torfzersetzung vermindert sich (25% statt 40% in der xerothermischen Zeit), nach der botanischen Zusammensetzung ist der Torf ein Schilf-Riedtorf. Im weiteren (Tiefe 40—50, 20—30 cm) findet eine neue Austrocknung bei endgültigem Verschwinden der Erle statt. Die Tiefe von 10—20 cm entspricht einer neuen Feuchtigkeitsperiode, offensichtlich schon der jetzigen; zu dieser Zeit nimmt die Menge des Weidenpollens auffallend zu und erreicht ihr Maximum im Torflager (84%); es steigt die Pollenmenge der Fichte, gleichzeitig damit und sogar etwas früher erscheint die Tanne.

In aller letzter Zeit (2—10 cm) gingen neue Veränderungen vor sich: es verschwindet die Tanne, es vermindert sich die Pollenmenge der Weide— das Bild ist gewissermassen charakteristisch für die Austrocknungsperiode, aber gleichzeitig findet eine Zunahme der Pollenmenge der Fichte statt, obgleich letztere gegenwärtig nicht in der Nähe des Torfmoors vorkommt. Augenscheinlich hängt die neue Veränderung der Verhältnisse in diesem Fall nicht von den Schwankungen des Klima ab, sondern wird durch die Entwicklung der Erosionsprozesse und anthropogene Ursachen bedingt; anderenfalls ist das Verschwinden der Tanne und Erle bei gleichzeitiger Zunahme der Pollenmenge der Fichte schwer zu erklären.

Somit bestehen die inselartigen Wälder, indem sie eine Reihe von Veränderungen sowohl quantitative als auch qualitative in ihrem Artbestand erlitten haben, schon lange. Die Erle, die während der ganzen Existenzdauer des Torfmoors zugegen war, verschwand vor gar nicht langer Zeit; die Birke und Weide erlebten eine Reihe von Verbesserungen und Verschlechterungen ihrer Existenzverhältnisse. Die Fichte existierte in den inselartigen Wäldern während des ganzen Abschnittes der geologischen Geschichte, welcher der Existenzdauer des Torfmoors entsprach. Das Torfmoor ist ohne Zweifel sehr alt; dies wird sowohl durch das Pollendiagramm selbst, als auch durch das unmittelbare Ablagern des Torfes auf den tertiären Sand bestätigt.

Zu Beginn der Nacheiszeit verbreitete sich die Holzvegetation offensichtlich bedeutend südlicher als der Fichtenwald von Naurzum in das Gebiet des Turgailandes. Für den relikten Charakter der inselartigen Wälder und ihre stärkere Verbreitung in der Vergangenheit spricht auch das Vorkommen in dieser Gegend, insbesondere in der Nähe des Fichtenwaldes von Naurzum, der relikten Pflanzen des Nordens. Zu diesen mehr oder weniger nördlichen Pflanzen gehören verschiedene Arten von *Sphagnum*, *Cladonia silvatica*, *Equisetum silvaticum*, *E. pratense*, *Comarum palustre*, *Parnassia palustris*, *Drosera rotundifolia*, *Solidago virga aurea*, *Betula pubescens*, zum Teil *Salix cinerea*.

Es muss bemerkt werden, dass in der ganzen Gegend zwischen dem Fichtenwald von Naurzum und dem Sypsyn-Agatsch Wald einerseits und den Fichtenwäldern Tersek-Karagai und Aman-Karagai andererseits relikte degradierte Böden, die wohl früher bewaldet waren, jetzt aber sich zu Steppen umwandeln, weit verbreitet sind.

National Park von Naurzum.
Kasachstan.

Eingegangen
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Übersetzt von A. Gurewitsch.

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MICROBIOLOGY

BIOCHEMICAL ACTIVITY OF THERMOPHILE BACTERIA

By A. A. IMŠENECKI

(Communicated by A. A. Richter, Member of the Academy, 18. XII. 1940)

Bacteria that make best growth at temperatures corresponding to those of protein coagulation in higher and most of the lower plants, have been known for a long time. Their activity is possible at the uppermost limit of the biokinetical temperature zone, and there is no doubt that once arisen by way of adaptation they can and do, if certain conditions are available, take active part in the mineralization of organic matter. On theoretical grounds some of their forms may be supposed to have formed out of mesophile bacteria, and if so, we can expect one and the same species to include two different strains with unequal temperature optimum. At the same time there may exist such primary thermophile bacteria as have no parallel forms among the known mesophiles. Speaking of «parallel» forms, I mean only their taxonomic position, for one and the same biochemical process can be called forth by organisms occupying different places in the system of bacteria.

Decomposition of organic matter at high temperatures gives ground to suppose among the thermophiles the existence of agents involved in most, if not in all, of the processes connected with transformations of carbon, nitrogen, sulphur and iron.

Entering upon my studies on thermophile bacteria, I took for starting point the following considerations. It is known that increase in temperature has an accelerating effect on physiological processes, and if the application of van't Hoff's law in microbiology is not possible without an array of corrections, nevertheless for many manifestations of the life activity of microorganisms the value of the temperature coefficient Q 10° varies between 1.5 and 4. This applies as well to multiplication and respiration of bacteria, as to certain processes of fermentation investigated along this line.

This intensification of physiological processes has been established with respect to mesophile species of bacteria and is not possible beyond certain limits, because the temperature maximum for the growth of these bacteria is comparatively low. In the case of thermophile bacteria proper the optimum temperature for development is 25—35° C higher compared to the mesophile. Theoretical calculations give a very considerable acceleration of processes as a result of this difference in temperature. It is therefore very likely that the thermophiles are endowed with a much higher biochemical activity than that of the mesophiles.

In order to verify this assumption I have followed up the physiological action of mesophile and thermophile bacteria in a series of comparative tests.

The results of these observations are reported in the present paper. In the papers that are to follow the characteristics of isolated thermophiles will be dealt with at a greater length.

The start was made by observations on proteolytic activity. A pure culture of a thermophile proteolytic bacteria and the mesophile species *Bac. mesentericus* were involved. They were seeded separately into a series of Erlenmeyer flasks, of which each has a volume of 50 cm³ and contained 15 cm³ of meat peptone gelatine. The thermophiles were grown at 60° C; the mesophiles at 25° C. After 24, 48 and 72 hours the contents of the flasks were titrated for carboxyl groups by means of an 0.05 *N* alcohol solution of alkali. The results of analyses are brought out by the curves in Fig. 1.

It will be obvious that the rate of proteolysis was much higher in the presence of thermophile bacteria than in the presence of *Bac. mesentericus*. In 48 and 72-hour tests the deviation, as is seen from the curves, was especial-

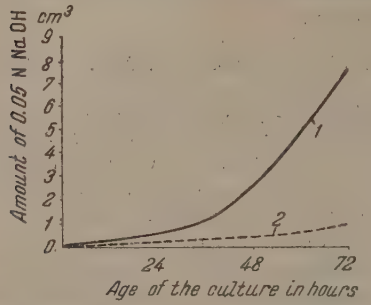

Fig. 1. Proteolysis in cultures of thermophile (*1*) bacteria and in cultures of *B. mesentericus* (*2*).

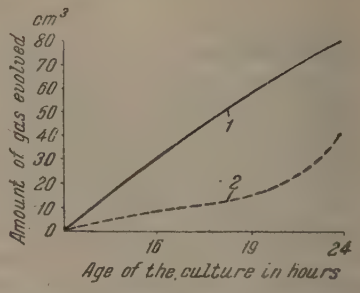

Fig. 2. Denitrification due to thermophile (*1*) and mesophile (*2*) bacteria.

ly strong, and the amount of alcohol solution of alkali consumed in titration was 6—8 times higher for the flasks with the thermophile cultures than for the flasks with *Bac. mesentericus*.

The rate of denitrification in the presence of mesophile and the respective thermophile bacteria was studied in the flasks containing Giltay medium added with soil and a fecal extract. The gas evolved was measured by volume 16, 19 and 24 hours after seeding. How strong was the activity of the thermophile denitrifying bacteria is evident from Fig. 2. In 24 hours they had reduced all the nitrates contained in the medium—neither nitrates nor nitrites could be detected there.

Our further step was to investigate the activity of amylolitic thermophile bacteria. Their cultivation was carried on at 60° C in a medium of the following composition: peptone, 0.5 g; insoluble starch, 3 g; chalk, 2 g; and a 20% potato decoction, 100 cm³. At the same time and in the same medium a culture of *Bac. mesentericus niger* was grown at 25° C. In preliminary tests I had found this strain, among 10 other strains of *Bac. mesentericus*, to convert starch into sugar the most intensively. The amount of sugar in the culture was determined after 24, 48 and 72 hours. In Fig. 3 are plotted the curves bringing out the amylolytic power of mesophile and thermophile bacteria. They show that in the presence of latter nearly 7 times more starch is converted into sugar in 48 hours.

Next followed the tests in which the rate of decomposition of cellulose by thermophile and mesophile bacteria was studied. Seeding was done into flasks containing the *VL* medium and a definite portion of filter paper care-

fully weighed out. As a thermophile bacteria was used a pure culture, strain 37/4. Its activity was compared against a mesophile culture which besides the cellulose form contained symbiotic anaerobic bacterium. The tests with thermophiles were run at 60° C, those with mesophiles at 37° C. The amounts of cellulose fermented in various intervals of time were determined by weighing (1).

The results are plotted in Fig. 4. It may be seen that the fermentation of cellulose was proceeding far more rapidly with the thermophiles than with mesophiles. In 8 days the former had destroyed 14 times as much cellulose as had the latter. The acceleration might have been more conspicuous still, if mesophile cultures with lower temperature optimum had been involved.

In 1923 Krohn had published his results obtained from studying the thermophile and mesophile bacteria. He had thoroughly investigated the assimilative ability of these bacteria with regard to various sources of nitrogen and carbon and established their response to various dyes permeating the membranes and protoplasm of their cells. But nothing typical in the behaviour of thermophiles had he been able to disclose in that way.

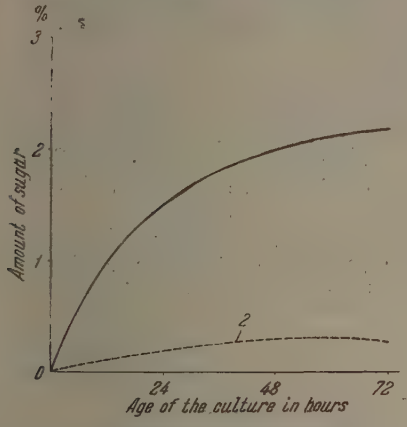

Fig. 3. Starch hydrolysis in the cultures of a thermophile amylolytic bacteria (*1*) and in cultures of *B. mesentericus niger* (*2*).

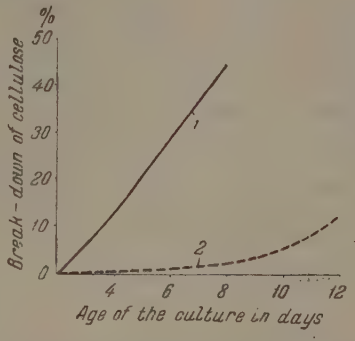

Fig. 4. Fermentation of cellulose in the presence of thermophile (*1*) and mesophile (*2*) bacteria.

What feature, indeed, should be pointed out as the most distinctive of these bacteria? In all of the works on morphology, physiology and distribution of thermophile organisms it is their response to temperature that has been considered as their specific character. Those species of bacteria which make best growth at 55—65° C are usually referred to thermophiles. This, of course, is a very formal kind of determination which will take no account of the changes in the physiology of the cell which arise, and correlate with those conditions of development which have been set up by the influence of high temperature. Naturally enough, the adaptation of thermophile bacteria is not controlled by the factor of temperature alone, but is also influenced by the medium, whose physical and chemical properties are bound to be greatly modified by so strong a shift in temperature. Of particular importance appears to be the decrease in the content of oxygen which is then less easily dissolved by the medium. It is perhaps for that reason that among the thermophiles it is more difficult to detect the bacteria responsible for oxidizing processes than e. g. the denitrifying or sulphate-reducing bacteria, or those responsible for carbohydrate fermentation. This is also in line with the

fact that with a good aeration of thermophile cultures there is observed an increased accumulation of enzymes, especially of amylase, in the medium.

Temperature has an influence upon the rate of life processes. In the logarithmic phase of growth the cells of thermophile bacteria multiply far more quickly compared to the mesophiles; the processes of assimilation and dissimilation are also considerably accelerated. In this respect a convincing evidence is furnished by our curves which show that proteolysis, reduction of nitrates, convertion of starch into sugar and fermentation of cellulose go on much more rapidly with the thermophiles than with the mesophiles in the medium. Especially strong was the difference in the rate of decomposition of cellulose, proteins and starch, while it was not so pronounced in the tests with denitrifying bacteria. The question as to whether or not the biochemical reactions so accelerated proceed at a rate corresponding to that calculated theoretically will be considered later, when we shall have made tests with the thermophile and mesophile strains of a single species.

So, from our consideration of the matter on points of physiology we have come to take the high speed of the processes called forth by the thermophile bacteria for their most typical feature. Apparently, no such acceleration can be produced by the mesophiles, even if the most favourable conditions for their development have been created.

So strong an intensification of life activity is bound to make the cells of thermophiles heavily stored with the products of metabolism, which cannot but produce a detrimental effect. Liberation from these products appears to be possible only to a certain limit, and it is perhaps because of this autointoxication consequent upon a high rate of life that the cells of the thermophile bacteria soon grow old.

While pointing out the high biochemical activity of thermophiles, I leave the question of its causal mechanism open. Whether this high level is reached by way of multiplication of bacteria or in some other way, is the problem to be solved by further research.

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EXPERIMENTELLE MORPHOLOGIE

NEUE METHODE DER VERLÄNGERUNG DER REGENERATIONSFÄHIGKEIT DER EXTREMITÄTEN BEI DEN ANUREN

Von L. W. POLEŽAJEW und I. I. MOROSOW

(Vorgelegt von I. I. Schmalhausen, Mitglied der Akademie, den 25. XI. 1940)

Die früheren Versuche haben gezeigt, dass man bei den Kaulquappen der Anuren die verlorene Regenerationsfähigkeit der Hinterextremitäten im Stadium IIa auf dem Wege der Traumatisation ([3],[4]) oder der Transplantation ([2],[8]) wiederherstellen kann. Die Mechanik der Wiederherstellung der Regeneration besteht in diesen Fällen in der Dedifferenzierung des Mesoderms (der Muskelanlagen, des Skeletts und der Bindegewebebildungen) im Organrest, die zu einem raschen Entstehen einer Masse isolierter Mesodermalzellen, die zusammen mit dem Epithel die Regenerationsanlage bilden ([8]), im Gebiet der Amputationswundoberfläche des Organs führt. Die Regenerationsanlage verwandelt sich auf dem Wege des Wachsens und der Differenzierung in eine gut ausgebildete Extremität. Ausserdem wurde gezeigt, dass man mit Hilfe der Reregenerationsmethode eine volle Regeneration der Hinterextremitäten bei den Kaulquappen in noch späteren Stadien hervorrufen kann und zwar in den Stadien IIb, IIIa, IIIb und sogar IV [über die Stadien s. Poležajew ([5])], wo der Prozess zu einem grossen Teil bereits beim jungen Frosch verläuft und wo die Extremitäten bei einer gewöhnlichen Amputation nicht mehr regenerieren können. Die Reregenerationsmethode ist die Methode der Regeneration des Regenerats, das wächst, sich formiert und an dem die Amputation ausgeführt wird, wenn das Tier sich auf einem späten Stadium der Metamorphose befindet, das Regenerat selbst, jedoch, morphologisch ein Stadium erreicht, in dem die Regeneration bei der normalen Extremität noch vollkommen möglich ist. Auch in diesem Falle hängt die Möglichkeit der Regeneration von dem Vorhandensein eines bestimmten Differenzierungsgrades der Extremität und besonders ihres mesodermalen Teiles ab.

Die Aufgabe der gegenwärtigen Arbeit bestand darin, eine Regeneration nicht des Regenerats, wie das in den Versuchen über die Reregeneration war, sondern des Organrestes zu erhalten und, zweitens, eine Regeneration in späteren Stadien als IIa zu erhalten, in welchen sich die Methoden der Traumatisation und Transplantation wirksam erweisen.

Als Prämissen für die Durchführung der gegenwärtigen Versuche dienten folgende Beobachtungen und Tatsachen: 1) das, dass die Wiederherstellung der Regenerationsfähigkeit der Extremitäten im Stadium IIa nach der Methode der Traumatisation und Transplantation stets durch das Eintreten einer Dedifferenzierung des Mesoderms des Organrestes bedingt ist; 2) das, dass bei der normalen Regeneration der Extremitäten bei den Kaulquappen im I Stadium, wo sie ihre Regenerationsfähigkeit noch nicht verloren haben, das Mesoderm des Organrestes stark dedifferenziert wird ([6],[7]); das Mesoderm

dedifferenziert sich auch bei der Regeneration bei Axolotln ([7]) und Amblystomen ([9]), was eine allgemeine Bedingung für die Regeneration komplizierter Organe bei den Wirbeltieren ist ([7]); und 3) das, dass bei der wiederholten Entfernung der Regenerationsanlage vom Schwanz der Kaulquappe die Regeneration nicht nur nicht gehemmt und nicht verlangsamt, sondern im Gegenteil, beschleunigt wird ([1]). In diesem Fall kann man annehmen, dass es sich um eine Steigerung der Dedifferenzierungsprozesse des Mesoderms des Organrestes handelt, was zur Beschleunigung der Bildung der Regenerationsanlage führt.

Die soeben angeführten Daten lassen die allgemeine Schlussfolgerung zu, dass die Beschädigung und der Regenerationsprozess selbst den Verlauf der Regeneration fördern, indem sie den Organrest mit Hilfe der Dedifferenzierung seiner Mesodermalgewebe, so zu sagen, verjüngen. Wir hofften, dass wir, indem wir im I Stadium der Metamorphose die Regeneration der Hinterextremitäten bei den Kaulquappen hervorriefen, den Organrest «verjüngen» würden, den es infolgedessen möglich sein würde zu veranlassen, auch in späteren Stadien zu regenerieren, wenn man die Regenerationsanlage, die sich gebildet hat, abschneidet.

Abb. 1. Hinterextremitäten der Kaulquappe: *a*—nach wiederholter Amputation; *b*—nach einer Amputation.

Unser Versuch war sehr einfach. Bei den Kaulquappen der *Rana temporaria* wurde im I Stadium das rechte Hinterbeinchen im proximalen Teil des Schienbeines amputiert. Nach 4—6 Tagen wurde das Regenerat, das das Stadium einer grossen Knospe oder eines Kegels erreicht hatte, abgeschnitten und zwar zusammen mit einer geringen Menge alter Gewebe. Gleichzeitig wurde auch auf dem gleichen Niveau das linke, Kontrollbeinchen derselben Kaulquappe, das seiner Entwicklung nach dem Metamorphosestadium I, IIa, IIb oder IIIa entsprach, amputiert. Ausser 14 auf den Frühstadien des Regenerationsprozesses fixierten Kaulquappen blieben zum Schluss des Versuches noch 123 Tiere. Die zweite Amputation erfolgte bei 10 von ihnen im Stadium I (1. Gruppe der Fälle), bei 62 im Stadium IIa (2. Gruppe), bei 40 im Stadium IIb (3. Gruppe) und bei 11 im Stadium IIIa (4. Gruppe).

In der ersten Gruppe regenerierten in allen Fällen beide Beinchen, wobei das rechte Beinchen immer schneller regenerierte als das linke, das Kontrollbeinchen, und die Differenzierung des rechten Beinchens (Abb. 1,*a*) stets weiter vorgeschritten war, als diejenige des linken (Abb. 1,*b*). Diese Erscheinung erklärt sich dadurch, dass die wiederholte Amputation des rechten Beinchens die Dedifferenzierung seines Mesoderms im Vergleich mit derjenigen des linken verstärkt, was zur Vergrösserung des Umfanges und zur Beschleunigung der Differenzierung seines Regenerats führt, was mit den Angaben von Filatow vollständig übereinstimmt ([10]).

In den Gruppen 2, 3 und 4 der Fälle hat das Kontrollbeinchen bereits vollständig oder teilweise seine Regenerationsfähigkeit verloren. Daher äusserte sich dort das Resultat der wiederholten Amputation des Versuchsbeinchens noch klarer, als in der Gruppe 1.

In der Gruppe 2 regenerierte das Versuchsbeinchen in 49 Fällen (79.03%) vollständig, während das Kontrollbeinchen gar nicht regenerierte; in 11 Fällen (17.74%) regenerierte das Versuchsbeinchen vollständig, das Kontrollbeinchen jedoch teilweise; in 1 Falle (1.61%) regenerierte das Versuchsbeinchen teilweise, das Kontrollbeinchen dagegen regenerierte nicht, und in 1 Falle (1.61%) regenerierten beide Beinchen nicht.

In der Gruppe 3 regenerierte das Versuchsbeinchen in 36 Fällen (90.0%) vollständig, während das Kontrollbeinchen nicht regenerierte (Abb. 2);

in 1 Falle (2.5%) regenerierte das Versuchsbeinchen vollständig, das Kontrollbeinchen jedoch teilweise; in 2 Fällen (5%) regenerierten beide Beinchen nicht, und in 1 Falle (2.5%) regenerierten beide Beinchen teilweise.

In der Gruppe 4 regenerierte das Versuchsbeinchen in 10 Fällen (90.9%) vollständig, während das Kontrollbeinchen nicht regenerierte, und in 1 Falle (9.1%) regenerierten beide Beinchen nicht.

Ein positives Resultat, d. h. das Überwiegen der Regeneration des Versuchsbeinchens über diejenige des Kontrollbeinchens, wurde in den Gruppen 2, 3 und 4 in 108 Fällen von 113, d. h. in 95.6% erhalten. Somit erwies sich die Methode der wiederholten Regeneration sehr wirksam in bezug auf die Wiederherstellung oder, richtiger gesagt, die Verlängerung der Regenerationsfähigkeit der Extremitäten bei den metamorphosierenden Kaulquappen.

Interessant ist, dass das positive Resultat der Regeneration und zwar die Zahl der Fälle, in denen das Versuchsbeinchen vollständig regenerierte, das Kontrollbeinchen aber nicht regenerierte, mit dem Alter der Versuchstiere zunimmt; nach der wiederholten Amputation betrug dasselbe in der Gruppe 2 im Stadium IIa—79.09%, in der Gruppe 3 im Stadium IIb—90.9% und im Stadium IIIa—90.9%. Dieses Resultat erklärt sich dadurch, dass bei relativ gleicher Effektivität der wiederholten Amputation in bezug auf das Versuchsbeinchen die Regenerationsfähigkeit bei den Kontrollbeinchen progressiv von den jüngeren Stadien (IIa) zu den älteren (IIIa) abnimmt. Hier muss jedoch erwähnt werden, dass nach der wiederholten Amputation die Regeneration in den älteren Stadien (IIIa) schlechter, gehemmter vor sich geht als in den früheren (IIa), was von dem hemmenden Einfluss des sich bei der Metamorphose des Tieres verändernden inneren humoralen Milieus auf das Wachstum des Regenerats abhängt.

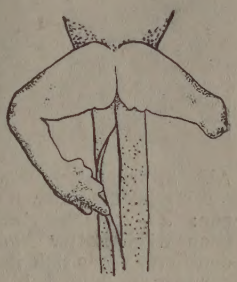

Abb. 2. Hinterextremitäten der Kaulquappe: die rechte regeneriert nach wiederholter Amputation, die linke regeneriert nicht nach der ersten Amputation.

Das fixierte Material wurde histologisch bearbeitet und mit Eisenhämatoxylin und Eosin gefärbt. Zur Fixierung wurden Tiere genommen: 1) im Moment der wiederholten Amputation mit einer Regenerationsanlage auf dem Versuchsbeinchen und einem normalen Kontrollbeinchen; 2) 2 Tage nach der wiederholten Amputation und 3) 4 Tage nach der wiederholten Amputation. Die Extremitäten regenerierten in 8—10 Tagen vollständig. Die mikroskopische Untersuchung des Versuchsmaterials ergab folgendes.

Zum Moment der wiederholten Amputation war das Mesoderm der Versuchsextremität stark dedifferenziert; das Gebiet der an der Amputationswundoberfläche anliegenden Muskelanlagen zerfällt fast vollständig in isolierte Zellen, das freie Ende des Skeletts schmilzt, der Grundstoff in demselben verschwindet, der Knorpel verwandelt sich in eine Mesenchymverdichtung, von der sich Mesodermalzellen abtrennen, die zusammen mit den Zellen, die sich aus den Muskelanlagen isoliert haben, in das Regenerationsblastem übergehen. In der Kontrollextremität sind das Skelett und die Muskelanlagen gut differenziert und inmitten derselben sind keinerlei isolierte Mesodermalzellen vorhanden.

Nach der wiederholten Amputation kann man in der Extremität zwei Zonen unterscheiden: den proximalen Teil des Organrestes, in dem die Gewebe fortfahren sich gut zu differenzieren, und den distalen, in dem die Mesodermalgewebe (Muskelanlagen und Skelett) sich stark dedifferenzieren, und die entstandenen isolierten Mesodermalzellen, indem sie allmählich auseinandergehen, in die mit einem Epithel bekleidete Regenerationsanlage über-

gehen (Abb. 3,*a*). Der Dedifferenzierungsgrad verändert sich hierbei sehr stark; bisweilen dedifferenziert sich der ganze distale Teil des Mesoderms des Organrestes, bisweilen aber, nur ein sehr geringer Teil desselben. Das hängt offenbar hauptsächlich von der Schnelligkeit der Metamorphose der Kaulquappe ab, in deren Verlauf die Mesodermalgewebe der Extremität sich stark differenzieren. In der Kontrollextremität dedifferenzieren sich die Mesodermalgewebe nicht, das Blastem bildet sich nicht und es erfolgt keine Regeneration (Abb. 3,*b*).

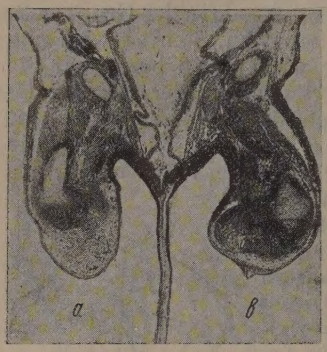

Abb. 3. Frontalschnitt der Hinterextremitäten der Kaulquappen: *a*—Versuchsextremitäten; nach wiederholter Amputation dedifferenzieren sich ihre Mesodermalgewebe und bildet sich die Regenerationsknospe; *b*—Kontrollextremität; nach der Amputation erfolgt keine Dedifferenzierung ihrer Mesodermalgewebe und keine Regeneration.

Die Möglichkeit der Regeneration hängt, somit, einerseits davon ab, ob sich die Mesodermalgewebe im Organrest dedifferenzieren, und andererseits von dem Einfluss des inneren humoralen Milieus auf dieselben, weil letzteres sich während der Metamorphose verändert und die Regeneration hemmt ([8]). Der Erfolg des Prozesses, d. h. ob eine Regeneration stattfindet oder nicht, hängt davon ab, welcher dieser beiden Faktoren seinen Einfluss früher äussert.

Schlussfolgerungen. 1) Ruft man den Regenerationsprozess an der einen der Hinterextremitäten der Kaulquappe hervor und amputiert man alsdann deren Regenerationsanlage und die andere, paarige, Kontrollextremität, so wurden folgende Erscheinungen beobachtet: a) ist im Moment der wiederholten Amputation die Regenerationsfähigkeit in der Kontrollextremität nicht verschwunden, so regeneriert die Versuchsextremität schneller und besser als die Kontrollextremität; b) ist im Moment der wiederholten Amputation die Regenerationsfähigkeit in der Kontrollextremität bereits verschwunden, so kann die Versuchsextremität trotzdem in den Stadien IIa, IIb und IIIa vollständig regenerieren, wobei man in den zwei letzten Stadien die Regeneration auf dem Wege der Traumatisation und Transplantation nicht hervorrufen kann.

2) Die wiederholte Amputation führt zur Verjüngung des Organs, zur Dedifferenzierung seiner Mesodermalgewebe, die infolge dessen ihre Regenerationsfähigkeit in den späteren Stadien bewahren, trotz des Einflusses des inneren, humoralen Milieus des Tieres, das sich im Prozess der Metamorphose verändert. Das letztere fördert das Fortschreiten der histologischen Differenzierung in der «nichtverjüngten» Kontrollextremität, im Zusammenhang womit ihre Regenerationsfähigkeit verloren geht.

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